



Solution
249

by Benjamin Bloch, Ph.D.

1- Area = $\frac{1}{2}$ base x height

From the apex drop a perpendicular to the base. The triangle being isosceles has equal base angles so that the perpendicular is also an angle bisector.

The height of the perpendicular bisector equals $s \cos \theta$, and the base equals $2s \sin \theta$

Therefore the area $A = s^2 \sin \theta \cos \theta$.

We want $dA/d\theta = 0$. Since $(d/d\theta) \sin \theta = \cos \theta$ and $(d/d\theta) \cos \theta = -\sin \theta$

We get $dA/d\theta = s^2 (\cos^2 \theta - \sin^2 \theta) = 0$.

Therefore $\cos \theta = \sin \theta$, $\theta = 45$ degrees, and thus the included angle is $2\theta = 90$ degrees.

2- $\sin 45 = \cos 45 = 2^{1/2}/2$, so that the maximum area = $\frac{1}{2} s^2$.

3- $P = s(2 + 2^{1/2})$.