Solution 248

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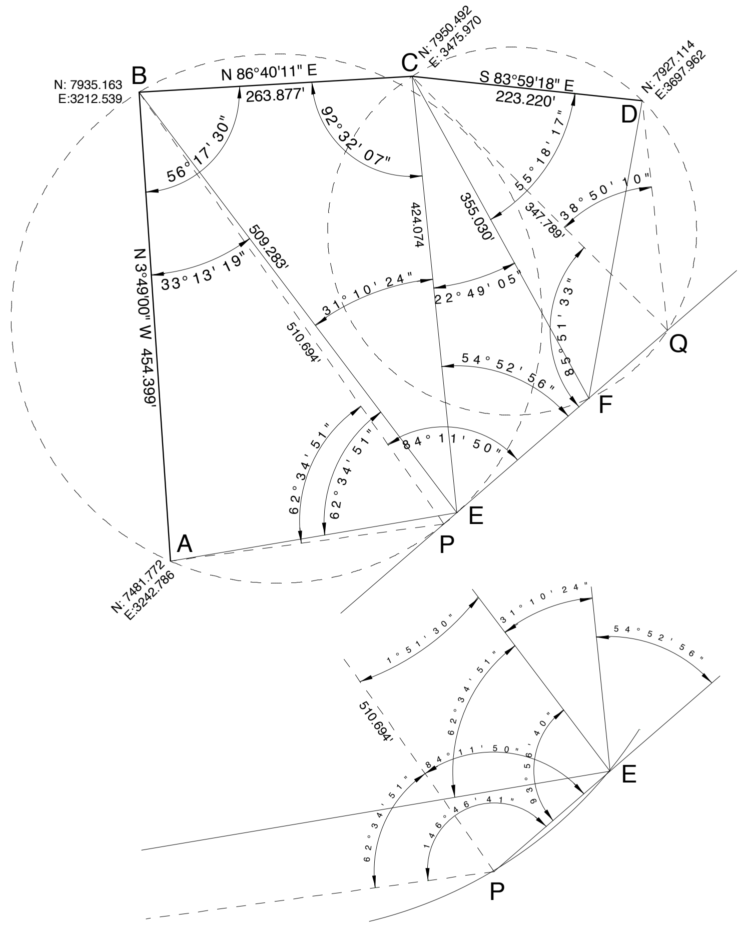

Inverse between points A-B, B-C and C-D: N 3°49'00" W 454.399', N 86°40'11" E 263.877' and S 83°59'18" E 223.220' respectively. To draw this to scale, construct a circle through the ends of chord A-B with $R = \frac{454.399}{2 \cdot \sin 62°34'51"} = 255.953$ (so any point on its circumference will subtend chord A-B with an angle of 62°34'51").

Construct B-E so angle ABE equals 33°13'19", the supplement of angle APQ, and point E lies on the circle. Angle EPA = 146°46'41" (because it subtends the chord A-E but on the opposite side of angle ABE).

Construct another circle through the ends of chord C-D with $R = \frac{223.220}{2 \cdot \sin 38°50'10"}$ =180.078 (so any point on its circumference subtends chord C-D with an angle of 38°50'10").

Construct C-F so angle DCF equals 55°18'17", the supplement of angle DQP, and point F lies on the circle. Extend line E-F to intersect the circles in points P and Q.

In triangle ABE:

$\frac{454.399}{\sin 62°34'51"} = \frac{EB}{\sin(180° - 33°13'19" - 62°34'51")}$, so EB = 509.283

In triangle CDF:

$\frac{223.220}{\sin 38°50'10"} = \frac{CF}{\sin(180° - 55°18'17" - 38°50'10")}$, and CF = 355.030

In triangle BCE:

$CE^2 = 263.877^2 + 509.283^2 - (2)(263.877)(509.283)\cos 56°17'30"$, so CE=424.074

Also: $\frac{\sin BEC}{263.877} = \frac{\sin BCE}{509.283} = \frac{\sin 56°17'30"}{424.074}$ so that angle BEC = 31°10'24" and angle BCE=92°32'07" (Don't forget to use the value over 90°).

In triangle CEF: $EF^2 = 424.074^2 + 355.030^2 - (2)(424.074)(355.030)\cos 22°49'05"$ from which EF = 168.322 and $\frac{\sin 22°49'05"}{168.322} = \frac{\sin CFE}{424.074} = \frac{\sin CEF}{355.030}$, so that angle CFE = 102°17'59" and angle CEF = 54°52'56"

Angle BAE=180° – 33°13'19" – 62°34'51"= 84°11'50", angle BPE=146°46'41" – 62°34'51" = 84°11'50" (which checks because they subtend the same chord, B-E).

In triangle BEP, angle BEP= 180° – 54°52'56" – 31°10'24 = 93°56'40" and angle PBE = 180° – 93°56'40" – 84°11'50" = 1°51'30"

and $\frac{PE}{\sin 1°51'30"} = \frac{509.283}{\sin 84°11'50"} = \frac{BP}{\sin 93°56'40"}$, so that PE = 16.600 and BP = 510.694

In triangle CFQ, angle CFQ = 72°42'01", angle CQF= 85°51'33" so angle FCQ=16°26'26"

$\frac{355.030}{\sin 85°51'33"} = \frac{FQ}{\sin 16°26'26"} = \frac{CQ}{\sin 77°42'01"}$, so FQ = 100.747 and CQ = 347.789

The azimuth of line B-P = 176°11'00"(azimuth of BA) – 33°13'19" + 1°51'30" = 144°49'11" and B to P is S 35°10'49" E 510.694', for point P = North 7517.751, East 3506.776

The azimuth of line CQ is 96°10'42"(azimuth of CD) + 55°18'17" – 16°26'26", and point Q is S 45°07'27" E 347.789' from point C, so point Q is North 7705.102, East 3722.426