Solution 247

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The area of the convex regular n-sided polygon of side s is given by

$A = \frac{1}{4} n s^2 \text{ ctn } 180/n$ while its perimeter $P_n = ns$.

For a circle, radius r, Area = $\pi r^2$ ; Perimeter = $P_1 = 2\pi r$

The two areas are equal so that $A = \frac{1}{4} n s^2 \text{ ctn } 180/n = \pi r^2$

Since $s = P_n/n$ and $r = P_1/2\pi$, upon substituting into the area equation:

$(n/4)\ (P_n^{\,2}/n^2) \text{ ctn } 180/n = \pi\ (P_1^{\,2}/4\pi^2)$

and simplifying, we get:

$P_n = (n/(\pi \text{ ctn } 180/n))^{1/2}\ P_1$

The equation relating the perimeter of any convex regular n-sided polygon of side s, whose area equals that of a circle of radius r, to the perimeter of the circle.

1. For n=3, $P_3 = (3/(\pi \text{ ctn } 180/3))^{1/2}\ P_1 = 1.286 P_1$ (n=3 an equilateral triangle)

   For n=4, $P_4 = (4/(\pi \text{ ctn } 180/4))^{1/2}\ P_1 = 1.128\ P_1$ (n=4 a square)

   For n=5, $P_5 = (5/(\pi \text{ ctn } 180/5))^{1/2}\ P_1 = 1.075\ P_1$ (n=5 a regular convex pentagon)

2. As n increases, $P_n$ approaches $P_1$ so that the many-sided convex regular polygon perimeter approximates the circumference of a circle of equal area.