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1. An equiangular quadrilateral has equal opposite sides having all right angles. [A rectangle of which a square is a special case.]

   Draw coordinates (r,θ) as shown in the diagram. Each horizontal length of the pool equals $2r \cos \theta$, and each vertical side equals $2r \sin \theta$. Thus, the area of the pool is $A = 4r^2 \cos \theta \sin \theta$.

   The variable angle θ sets the shape of the circle-inscribed pool. We are seeking the value of θ, which will make the pool area a maximum.

   One can intuit that when $\cos \theta = \sin \theta$ that maximum will be reached.
   Thus, $\theta = 45$ degrees.

   Or one can use calculus to get the analytic solution.
   We want $dA/d\theta = 0$. Now $d(4r^2 \cos \theta \sin \theta)/d\theta = 4r^2(-\sin^2\theta + \cos^2\theta) = 0$
   Solving for θ yields $\sin \theta = \cos \theta$, so that $\theta = 45$ degrees.

The area of the pool is thus $A = 4r^2 \cos 45 \sin 45 = 2r^2$

2. The shape of the pool is a square.

3. The outside pool area within the circle is $\pi r^2 - 2r^2 = r^2(\pi - 2)$.