Solution 238

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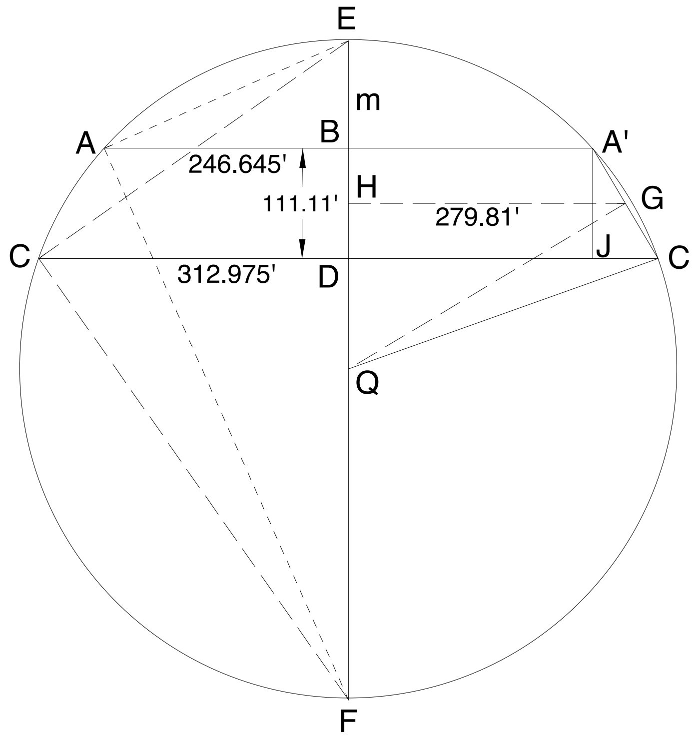

Draw diameter EF, bisecting both chords at right angles at points B and D.
AB = ½ (493.29) = 246.645 and CD = ½ (625.95) = 312.975

$\frac{AB}{BF} = \frac{m}{AB}$, and BF = 2·r – m ∴

$\overline{AB}^2 = (2 \cdot r - m) \cdot m = 2 \cdot r \cdot m - m^2$ .... [1]

$\frac{CD}{DF} = \frac{ED}{CD}$, and ED = m + 111.11 and DF = 2·r – 111.11 – m ∴

$$\frac{CD}{2r - 111.11 - m} = \frac{m + 111.11}{CD}$$

$$\begin{aligned}\overline{CD}^2 &= (2 \cdot r - 111.11 - m)(m + 111.11)\\ &= 2 \cdot r \cdot m + 222.22 \cdot r - 111.11 \cdot m - 111.11^2 - m^2 - 111.11 \cdot m\\ &= (2 \cdot r \cdot m - m^2) + 222.22 \cdot r - 222.22 \cdot m - 111.11^2\\ &= \overline{AB}^2 + 222.22 \cdot r - 222.22 \cdot m - 111.11^2\end{aligned}$$

$312.975^2 = 246.645^2 + 222.22 \cdot r - 222.22 \cdot m - 111.11^2$

$m = \frac{222.22 \cdot r - 49465.0267}{222.22}$, and r – 222.595 = m

Substituting into [1], $246.645^2 = 2 \cdot r\,(r - 222.595) - (r - 222.595)^2$

$60{,}833.756 = 2 \cdot r^2 - 445.19 \cdot r - (r^2 - 445.19 \cdot r + 49548.534)$ from which
$r^2 = 110{,}382.29$ and $r = 332.238$

Alternatively,

Let the perpendicular bisector of A'C' pass through G and the radius point, Q.
Let GH be perpendicular to EF and A'J perpendicular to DC'.

Angle JA'C' is $\tan^{-1} = \frac{(312.975 - 246.645)}{111.11} =$ 30°50'10", as is angle HGQ.

HQ= 279.81·tan30°50'10"=167.039.

DQ=167.039 – ½ (111.11) = 111.484

The radius, QC'= $\sqrt{(DQ)^2 + (DC')^2}$ =332.238