Solution

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Let point A be the center of the circle of radius 49', point D the center of the circle with radius 68', point F the center of the circle with radius 80' and point C the center of the dashed line circle with radius R.

$$AC = \sqrt{R^2 - 49^2},\ DC = \sqrt{R^2 - 68^2} \text{ and } FC = \sqrt{R^2 - 80^2}$$

In triangle CAB, AB=190.65 – C_N, and BC= C_E – 70.22, where C_N is the northing of point C and C_E is the easting of point C.

In triangle CED, CE = 256.65 – C_N and ED = 181.72 – C_E and in triangle CFG, CG=C_N – 130.65 and GF= 223.22 – C_E

$R^2 - 49^2 = (190.65 - C_N)^2 + (C_E - 70.22)^2$ .... (1)
$R^2 - 68^2 = (256.65 - C_N)^2 + (181.72 - C_E)^2$ .... (2)
$R^2 - 80^2 = (C_N - 130.65)^2 + (223.22 - C_E)^2$ .... (3)

Expanding and rearranging:

$R^2 = 43{,}679.27 + C_N^2 - 381.3C_N + C_E^2 - 140.44C_E$ .... (1)
$R^2 = 103{,}515.38 + C_N^2 - 513.3C_N + C_E^2 - 363.44C_E$ .... (2)
$R^2 = 73{,}296.59 + C_N^2 - 261.3C_N + C_E^2 - 446.44C_E$ .... (3)

Subtracting (2) from (1):

$0 = -59{,}836.11 + 132\ C_N + 223C_E$ .... (4)

Subtracting (3) from (2):

$0 = 30{,}218.79 - 252C_N + 83C_E$ .... (5)

Equating (4) and (5):

$C_E = 643.2493 - 2.742857C_N$ .... (6)

Substituting (6) back into (5):

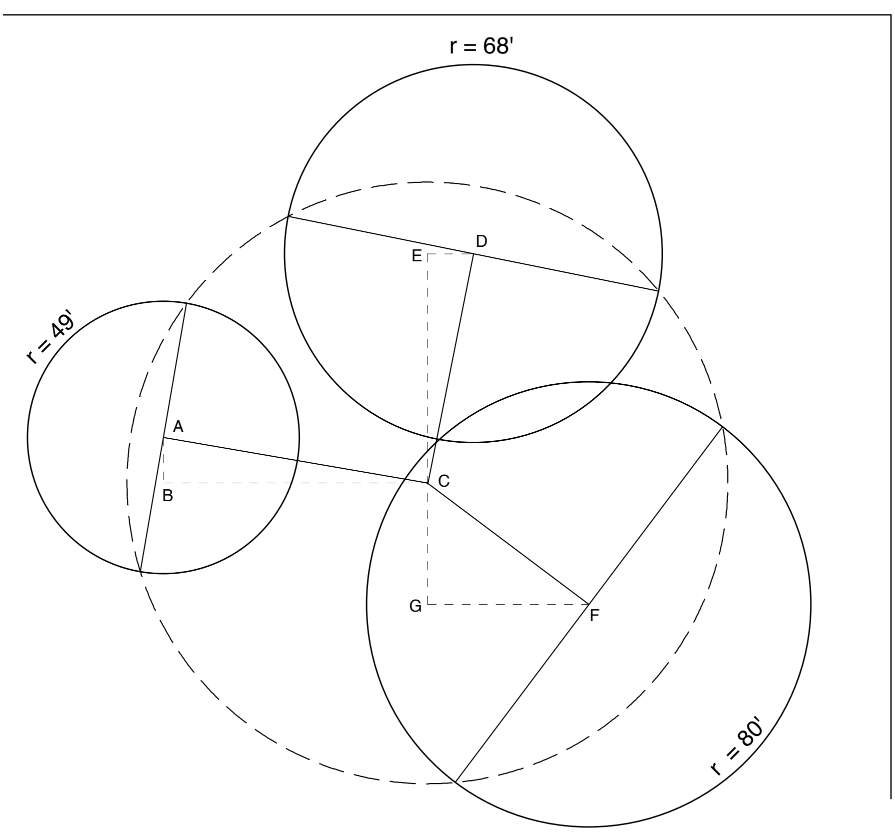
$0 = 30{,}218.79 - 252C_N + 83(643.2493 - 2.742857C_N)$ and $C_N = 174.31$

Substituting the value of C_N back into (6), $C_E = 165.145$

AB = 190.65 – 174.31 = 16.34
BC = 165.145 – 70.22 = 94.925

$$AC = \sqrt{16.34^2 + 94.925^2} = 96.32$$

$$R = \sqrt{AC^2 + 49^2} = 108.068$$



(If you would like to see an interactive visualization of this problem where you can change the circle locations and radii, and the fourth circle moves in real time, go to mathworld.com and click on Demonstrations Project. Type in “circle bisecting three circles.” To see all of Dr. Jaime Rangel-Mondragon’s fascinating contributions, type in his name.)