Solution

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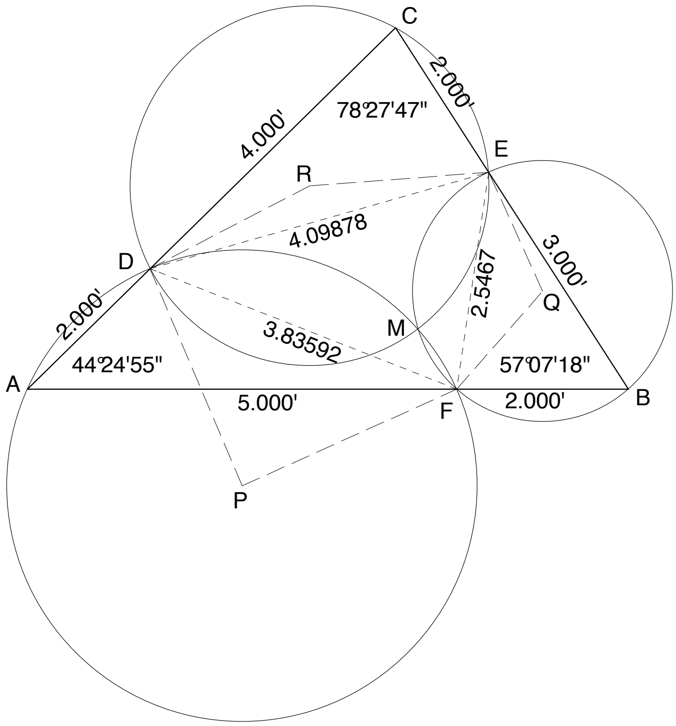

First, calculate the angles of triangle ABC:

$$\cos A = \frac{\overline{AC}^2 + \overline{AB}^2 - \overline{BC}^2}{2 \cdot \overline{AC} \cdot \overline{BC}} = \frac{6^2 + 7^2 - 5^2}{2 \cdot 6 \cdot 7} = 0.714285714, \sphericalangle A = 44°24'55"$$

$$\cos B = \frac{\overline{BA}^2 + \overline{BC}^2 - \overline{AC}^2}{2 \cdot \overline{BA} \cdot \overline{BC}} = \frac{7^2 + 5^2 - 6^2}{2 \cdot 7 \cdot 5} = 0.542857143, \sphericalangle B = 57°07'18"$$

$$\cos C = \frac{\overline{CA}^2 + \overline{CB}^2 - \overline{AB}^2}{2 \cdot \overline{CA} \cdot \overline{CB}} = \frac{6^2 + 5^2 - 7^2}{2 \cdot 6 \cdot 5} = 0.20000, \sphericalangle C = 78°27'47"$$

Chords DF, DE and EF can be calculated by the Law of Cosines:

$\overline{DF}^2 = \overline{AD}^2 + \overline{AF}^2 - 2 \cdot \overline{AD} \cdot \overline{AF} \cos \sphericalangle A = 2^2 + 5^2 - 2 \cdot 2 \cdot 5 \cdot 0.714285714 = 14.714285$

DF=3.83592

$\overline{DE}^2 = \overline{CD}^2 + \overline{CE}^2 - 2 \cdot \overline{CD} \cdot \overline{CE} \cos \sphericalangle C = 4^2 + 2^2 - 2 \cdot 4 \cdot 2 \cdot 0.2 = 16.8000$

DE=4.09878

$\overline{EF}^2 = \overline{BE}^2 + \overline{BF}^2 - 2 \cdot \overline{BE} \cdot \overline{BF} \cos \sphericalangle B = 3^2 + 2^2 - 2 \cdot 3 \cdot 2 \cdot 0.542857143 = 6.4857$

EF=2.5467

Denote the radius point of circle A-D-M-F as "P", the radius point of circle B-F-M-E as "Q", and the radius point of circle C-E-M-D as "R". Draw the radial lines from "P" to chord DF, from "Q" to chord FE and from "R" to chord DE.

$\sphericalangle DPF = 2 \cdot \sphericalangle A, \sphericalangle FQE = 2 \cdot \sphericalangle B, and \sphericalangle DRE = 2 \cdot \sphericalangle C$.

With the chord length and central angle known the radius can be calculated from the chord formula rearranged:

$$\frac{DF}{2 \cdot \sin A} = R_P, \frac{EF}{2 \cdot \sin B} = R_Q, \frac{DE}{2 \cdot \sin C} = R_R$$

$$R_P = \frac{3.83592}{2 \cdot \sin 44°24'55"} = 2.7405$$

$$R_Q = \frac{2.5467}{2 \cdot \sin 57°07'18"} = 1.5162$$

$$R_R = \frac{4.09878}{2 \cdot \sin 78°27'47"} = 2.09165$$

Note: By calculating coordinates for points D, E and F, the chords DF, DE and EF could have been determined by inverse.