Solution 204

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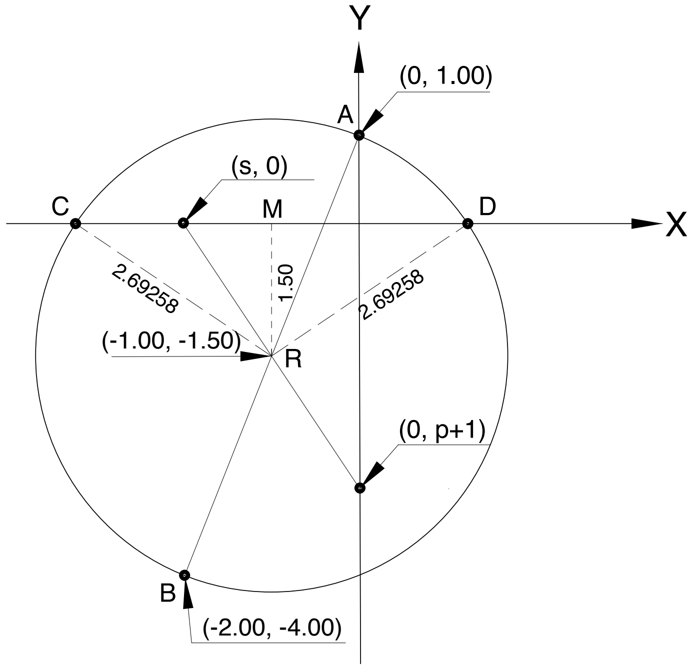

The values of C and D are readily calculated, first by finding the coordinate of the radius point, R, which is the midpoint of AB and the average of its coordinates: (–1.0, –1.5).

The radius is then $r=\sqrt{(1+1.5)^2+1.0^2}=\sqrt{7.25}=2.69258$

(Alternatively it is $\frac{1}{2}\sqrt{(1+4)^2+2^2}=\frac{\sqrt{29}}{2}=2.69258$)

Construct a perpendicular from R to the x-axis at point M. RM=1.5.

$CM=MD=\sqrt{2.69258^2-1.5^2}=\sqrt{5}=2.23607$

$C=-1.0-\sqrt{5}=-3.23607, D=-1.0+\sqrt{5}=1.23607$

The general formula for a circle centered at (h, k) is $(x-h)^2+(y-k)^2=r^2$; for this circle it is $(x+1)^2+(y+1.5)^2=r^2=7.25$

When y= 0, $x^2+2x+1+2.25-7.25=0$, or $x^2+2x-4=0$

From the quadratic equation: $x=\frac{-2\pm\sqrt{2^2+4\cdot 4}}{2}=\frac{-2\pm\sqrt{20}}{2}=-1\pm\sqrt{5}$

The x values of C and D are therefore the roots of the quadratic equation $x^2+2x-4=0$, or $x^2-(-2)x+(-4)=0$. Note the values (-2, -4) of point B.

In any quadratic equation of the form $x^2-sx+p=0$, a circle diameter through (0, 1) and (s, p) is called the Carlyle Circle of the equation. Where the circle crosses the x-axis is/are the root(s) of the equation. It is the CONSTRUCTION of the roots.

The center of the circle is also the midpoint of the line joining (s, 0) and (0, p+1), or $\left(\frac{s}{2},\frac{p+1}{2}\right)$

If m and n are the roots of the equation, it may be written as

$x^2-sx+p=(x-m)(x-n)=x^2-(m+n)x+mn$

so that s = (m + n) and p = mn. (These are known as Vieta's equations.)

In this case, $s=(-1+\sqrt{5})+(-1-\sqrt{5})=-2$ and $p=(-1+\sqrt{5})(-1-\sqrt{5})=-4$