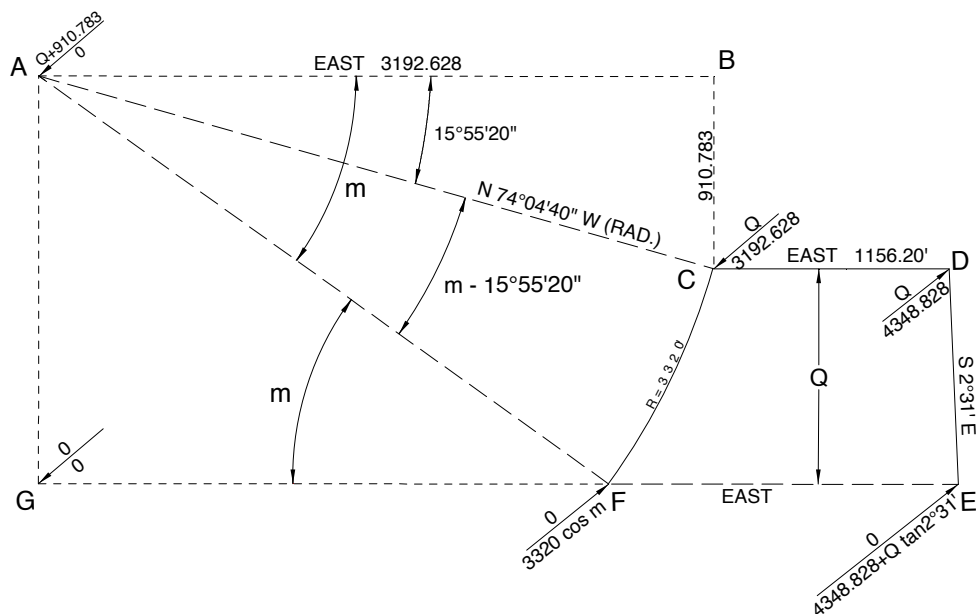


Solution 200

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First, rotate the figure so line C-D is East. Draw A-B parallel with C-D so C-B is perpendicular to A-B. Draw E-F-G parallel with C-D at distance Q southerly so A-G is perpendicular to G-F-E. A-B is then $(3320)(\cos 15^\circ 55' 20'') = 3192.628$ and B-C is $(3320)(\sin 15^\circ 55' 20'') = 910.783$

Let Point G be North: 0, East: 0. The coordinate of Point A is then N: $Q+910.783$, E: 0. Point C is N: Q, E: 3192.628. Point D is N: Q, E: 4348.828. Point E is N: 0, E: $3192.628+1156.20+Q \cdot \tan 2^\circ 31'$ and Point F is N: 0, E: $3320 \cos m$. Then, by the method of Problem No. 92, twice the area of ACDEF will be:

$$(Q+910.783)(3192.628) + 4348.828 Q + Q(4348.828 + Q \tan 2^\circ 31') - 3192.628 Q - (3320 \cos m)(Q+910.783)$$

And the area will be:

$$0.021976195 Q^2 + 4348.828 Q - 1660(\cos m)(Q+910.783) + 1453895.65$$

(The values of zero for the N'ing and E'ing become very apparent here!)

Subtracting the sector A-C-F should yield 36 acres, or 1,568,160 sq. ft.

$$\text{The sector area is } (\pi/360)(m-15.922222222)(3320^2)$$

There are two variables, m & Q. $Q = 3320 \sin m - 910.783$ and m can be expressed as a function of Q by the relation $\cos^2 m = 1 - \sin^2 m$, where $\sin m = (Q+910.783)/3320$, but m is still by itself in the sector equation.

It is easier to make an educated guess of m and solve iteratively, viz.:

Since m must be larger than $15^\circ 55' 20''$, start with $m = 20^\circ$. Q is then 224.7239.

And the area calculates to be 6.17056 acres. Letting $m = 30^\circ$, Q is then 749.217 and the area calculates to be 22.58738 acres. Letting $m = 40^\circ$, Q is 1223.272 and the area calculates to be 40.79054 acres, and we have found the range in which m falls.

Interpolating linearly between $m=30^\circ$ and $m=40^\circ$, we are presented with $m=37^\circ 22' 09''$, from which $Q=1104.291$ and the area 35.851222 acres. Another linear interpolation between $m=30^\circ$ and $m=37^\circ 22' 09''$ yields $m=37^\circ 26' 55''$ for a $Q=1107.938$ and an area of 35.99861 acres. A final linear interpolation using $m=37^\circ 22' 09''$ and $m=37^\circ 26' 55''$ gives a value of $m=37^\circ 26' 57''$, a $Q=1107.973$ and an area of 36.00000053 acres.

Arc CF is subtended by $21^\circ 31' 37''$ and has an arc length of 1,247.38'.

$$DE = [(3320)(\sin 37^\circ 26' 57'') - 910.783] / \cos 2^\circ 31' = 1,109.04'$$

$$EF = 3192.628 + 1156.20 + 1107.973 \cdot \tan 2^\circ 31' - (3320)(\cos 37^\circ 26' 57'') = 1,761.80'$$