Solution

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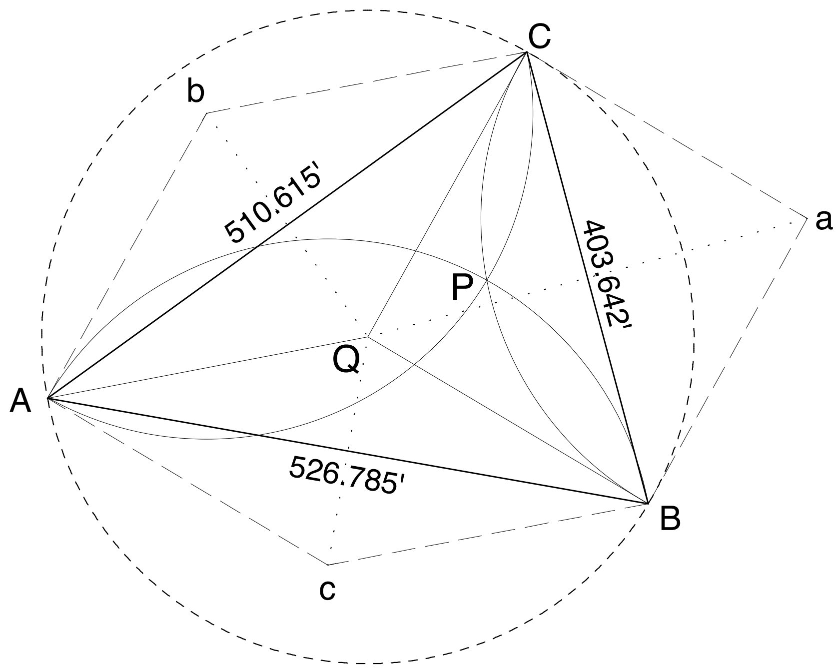

Calculate the angles of the triangle by the Law of Cosines:

$$\cos A = \frac{AC^2 + AB^2 - BC^2}{2 \cdot AC \cdot AB} = \frac{510.615^2 + 526.785^2 - 403.642^2}{2(510.615)(526.785)}, \text{ A = 45°45'46.1"}$$

$$\cos B = \frac{403.642^2 + 526.785^2 - 510.615^2}{2(403.642)(526.785)}, \text{ B = 65°00'12.4"}$$

$$\cos C = \frac{403.642^2 + 510.615^2 - 526.785^2}{2(403.642)(510.615)}, \text{ C = 69°14'01.5"}$$

Construct the circumcircle of triangle A-B-C. The radius point, Q, is at the intersection of the perpendicular bisectors of the chords, the triangle sides.

$$\sphericalangle AQC = 2 \cdot \sphericalangle B, \sphericalangle AQB = 2 \cdot \sphericalangle C, \sphericalangle CQB = 2 \cdot \sphericalangle A$$

$$r = \frac{chord}{2\sin(\Delta/2)} = \frac{510.615}{2\sin 65^\circ 00'12.4"} = \frac{403.642}{2\sin 45^\circ 45'46.1"} = \frac{526.785}{2\sin 69^\circ 14'01.5"} = 281.693$$

Construct the centers (a, b and c) of the three arcs. The centers lie at the intersection of the perpendicular bisectors of the chords PC, PB and PA (not shown).

The lines Ab, bC, Ca, aB, Bc and cA are all equal to the required radius. Since each radius point is perpendicular to the midpoint of any chord, b is perpendicular to the midpoint of AC, a is perpendicular to the midpoint of CB and c is perpendicular to the midpoint of AB. But Q is also perpendicular to the same point, making AbCQ, QCaB and BcAQ a rhombus, since a rhombus is the only quadrilateral that has diagonals that are perpendicular to (and bisect) each other. That makes Ab = QC, bC = AQ, Ca = QB, aB = CQ, Bc = AQ and cA = BQ, but QB = QA = QC = 281.693, the radius of the circumcircle.

The radius of each arc is therefore 281.693.

(The arcs are known as Johnson Circles. The circumcircle of the triangle formed by points a, b and c has its center at P.)