Solution
# 192

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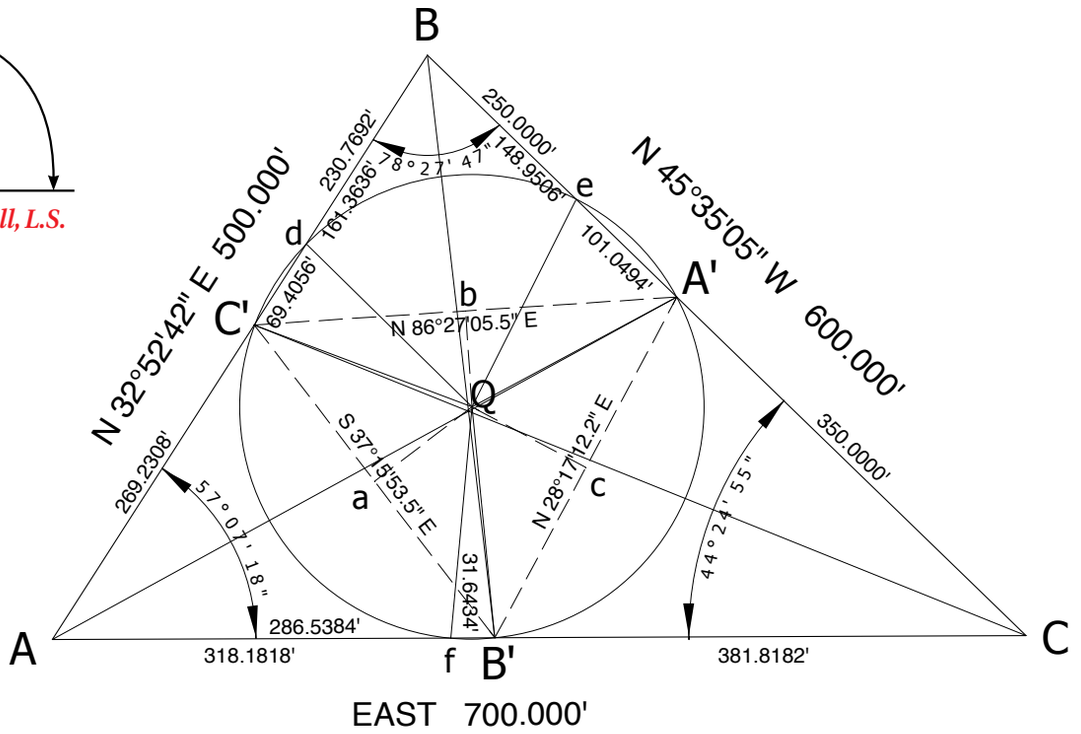


By the Law of Cosines:

Angle A = 57°07'18", angle B = 78°27'47", and angle C = 44°24'55"
Angle C-A-A' = angle A'-A-B = 28°33'39"
Angle A-B-B' = angle B'-B-C = 39°13'53.5"
Angle B-C-C' = angle C'-C-A = 22°12'27.5"

Bisectors divide the intersected side into parts proportional to the other sides:

$$\frac{AC'}{500} = \frac{700}{700+600}, AC' = 269.2308, \quad \frac{C'B}{500} = \frac{600}{600+700}, C'B = 230.7692,$$

$$\frac{BA'}{600} = \frac{500}{500+700}, BA' = 250.0000, \quad \frac{A'C}{600} = \frac{700}{700+500}, A'C = 350.0000,$$

$$\frac{AB'}{700} = \frac{500}{500+600}, AB' = 318.1818, \quad \frac{B'C}{700} = \frac{600}{500+600}, B'C = 381.8182$$

Letting point A be North 0, East 0, and line AC = East, point C is North 0, East 700 and point B, by bearing-bearing, azimuth-azimuth or distance-distance intersection is North 419.9125, East 271.4286

By traversing or proportional parts, C' is North 226.1067, East 146.1537, A' is North 244.9488, East 450.0000, and B' is North 0, East 318.1818

The radius point of the circle is NOT the intersection of the triangle bisectors. C'A', A'B' and B'C' are chords of the circle. Their perpendicular bisectors intersect at the center of the circle.

The coordinate of the midpoint of any line is the average of the end coordinates, so that "a" is North 113.0534, East 232.1678, point "b" is North 235.5278, East 298.0768, and point "c" is North 122.4744, East 384.0909. The bearing of B'C' is N 37°15'53.5" W, B'A' is N 28°17'12.2" E, and C'A' is N 86°27'05.5" E. Using points "a" and "c", and calling the radius point Q, the bearing of "ac" is N 86°27'05.5" E (same as C'A') and the perpendicular bisectors at "a" and "c" are N 52°44'06.5" E and N 61°42'47.8 W, respectively. A sketch will show

$$\frac{aQ}{\sin 31°50'06.7"} = \frac{cQ}{\sin 33°42'59.0"} = \frac{152.2149}{\sin 114°26'54.3"}$$

from which aQ = 88.1983 and cQ = 92.8140

so that point Q is North 166.4575, East 302.3601

QA' = N 62°00'11" E 167.2077, QB' = S 5°25'47" E 167.2077 and N 69°06'00" w QC' = 167.2078 = radius of circle

In triangle QC'd, angle dC'Q = 78°01'18", angle C'Qd = 23°57'24" and chord C'd = 69.4056

In triangle QA'e, angle QA'e = 72°24'44 angle A'Qe = 35°10'32" and chord A'e = 101.0494

In triangle B'Qf, angle fB'Q = 84°34'13", angle B'Qf = 10°51'34" and chord B'f = 31.6434

Except for round-off errors, chord A'e = chord B'f + chord C'd.