Solution
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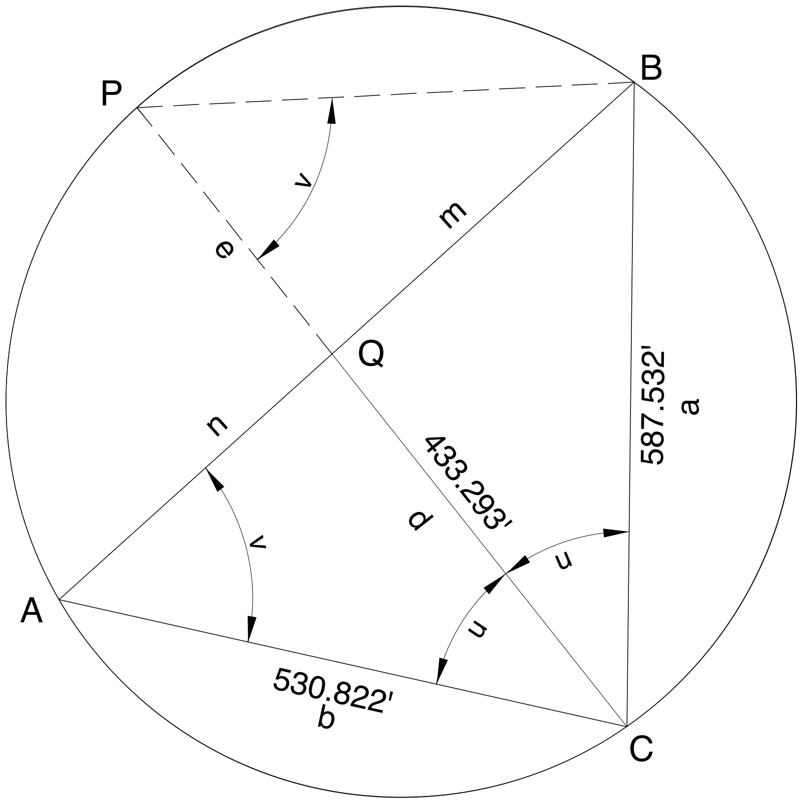

Construct the circumcircle of the triangle. Extend CQ to P. Draw PB.

Let AC = b, BC = a, AQ = n, QB = m, PQ = e and QC = d.

Triangle BCP is similar to triangle QCA; therefore, $\frac{b}{d} = \frac{d+e}{a}$

$ab = d(d+e) = d^2 + de$

$d \cdot e = m \cdot n$, because the products of intersecting chord segments are equal.

$d^2 = ab - mn$

In this case, $433.293^2 = (587.532)(530.822) - mn$, or $mn = 124{,}132.0875$

$m^2 = a^2 + d^2 - 2 \cdot a \cdot d \cdot \cos u$ and $n^2 = b^2 + d^2 - 2 \cdot b \cdot d \cdot \cos u$

$$\frac{a^2 + d^2 - m^2}{2ad} = \frac{d^2 + b^2 - n^2}{2db}$$

$$b(a^2 + d^2 - m^2) = a(d^2 + b^2 - n^2)$$

$$a^2b + bd^2 - bm^2 = ad^2 + ab^2 - an^2$$

$$an^2 = ad^2 + ab^2 - a^2b - bd^2 + b \cdot \left(\frac{124{,}132.0875}{n}\right)^2$$

$$n^2 = d^2 + b^2 - ab - \frac{bd^2}{a} + \frac{b}{a} \cdot \frac{124{,}132.0875^2}{n^2}$$

$$n^4 - (d^2 + b^2 - ab - \frac{bd^2}{a}) \cdot n^2 - \frac{124{,}132.0875^2 \cdot b}{a} = 0$$

$$n^2 = \frac{-11{,}981.52727 \pm \sqrt{11{,}981.52727^2 + (4)13{,}921{,}483{,}155.2}}{2}$$

$$n^2 = \frac{-11{,}981.52727 \pm 236{,}282.6477}{2}$$

from which n = 334.889 and m = 370.666

AB = n + m = 705.555