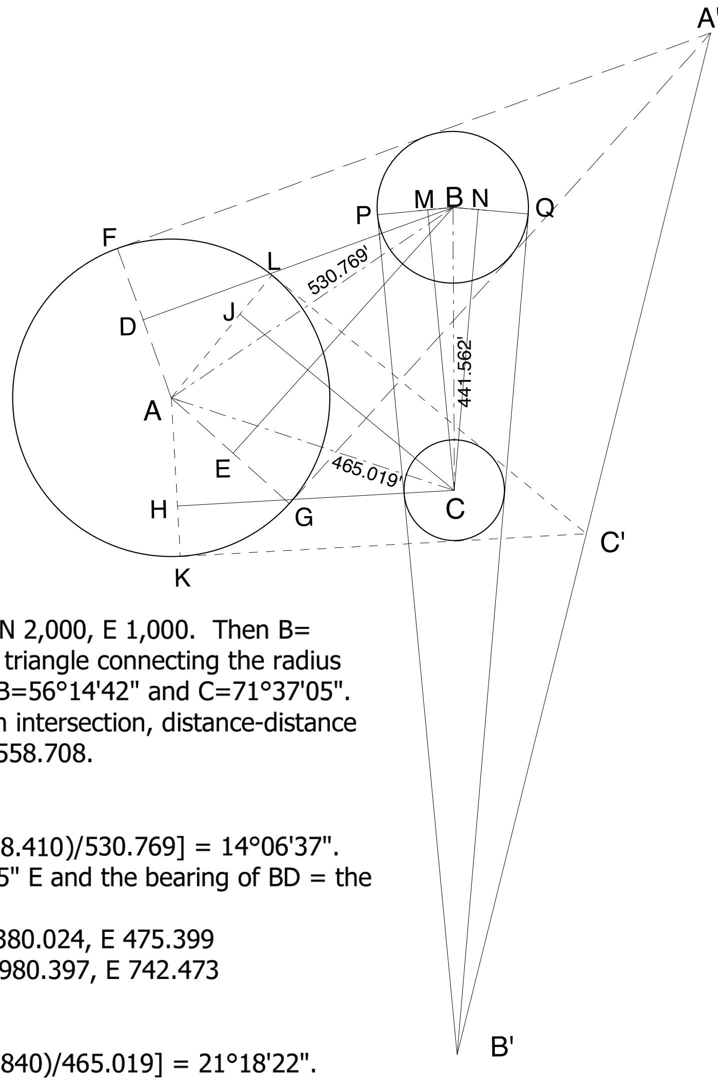
Solution 164

by Dave Lindell, L.S.



Let CB be North. Assign a coordinate to C, say, N 2,000, E 1,000. Then B= N 2441.562, E 1000. Solve for the angles in the triangle connecting the radius points using the Law of Cosines: A=52°08'13", B=56°14'42" and C=71°37'05". By bearing-bearing intersection, azimuth-azimuth intersection, distance-distance intersection or by traversing, A= N 2146.643, E 558.708.

Draw BD and BE so AD ⊥ BD and AE ⊥ BE.
Angle ABD = angle ABE = arcsin [(247.805 – 118.410)/530.769] = 14°06'37".
The bearing of BE = bearing of A'G = N 42°08'05" E and the bearing of BD = the bearing of FA' = N 70°21'19" E.
From A, F is N 19°38'11 W 247.805', so F = N 2380.024, E 475.399
G is S 47°51'55" E 247.805', so G = N 1980.397, E 742.473

Draw CH and CJ so AH ⊥ CH and AJ ⊥ CJ.
Angle ACJ = angle ACH = arcsin [(247.805 – 78.840)/465.019] = 21°18'22".
The bearing of CH = bearing KC' = N 87°04'33" E and the bearing of CJ = the bearing of LC' = S 50°18'43" E.
From A, K is S 2°55'27" E 247.805', so K = N 1899.161, E 571.350
L is N 39°41'17" E 247.805', so L = N 2337.337, E 716.958

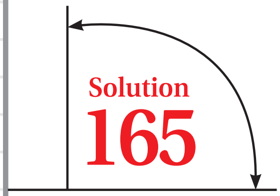
Draw CM and CN so BM ⊥ CM and BN ⊥ CN.
Angle BCM = angle BCN = arcsin [(118.410 – 78.840)/441.562] = 5°08'29".
The bearing of CM = bearing of PB' = S 5°08'29" E and the bearing of BN = the bearing of QB' = S 5°08'29" W.
From B, P is S 84°51'31" W 118.410', so P = N 2430.951, E 882.066
Q is S 84°51'31" E 118.410', so Q = N 2430.951, E 1117.934

By bearing-bearing or azimuth-azimuth intersection,
A' = N 2711.439, E 1403.826
C' = N 1931.574, E 1205.909
B' = N 1120.225, E 1000.000

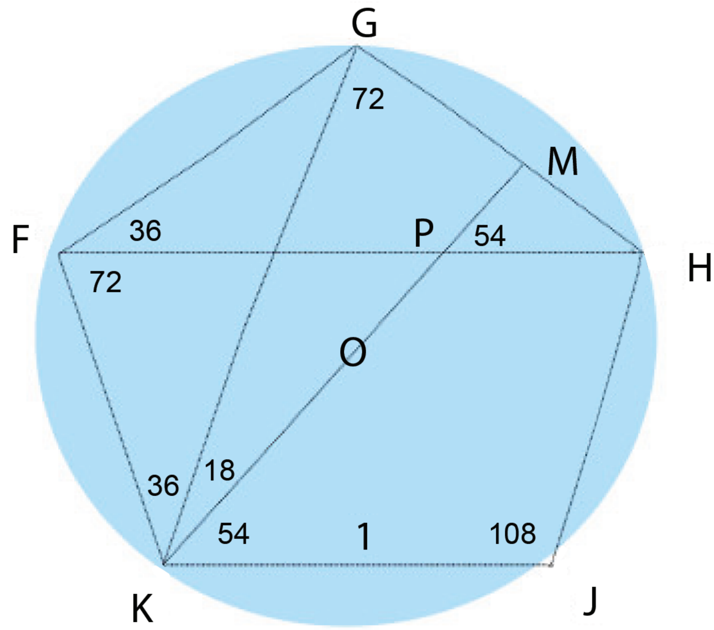
Bearing B'C' = N 14°14'25" E, bearing C'A' = N 14°14'24" E.

(Note: bearing B'A' = N 14°14'25" E, all to the nearest second.)

For ANY three circles, none of which are completely inside another, the intersections of the common external tangents will always lie on a straight line. This is Monge's Circle Theorem.



by Benjamin Bloch, Ph.D.



1- The regular pentagon divides the circle into 5 equal arcs of 360°/5 = 72° each. ∢ HJK subtends arc KFGH which is 3 x 72° = 216° and thus ∢ HJK is 216°/2 = 108°, (as are all the pentagon ∢s).
∢ FKG subtends arc FG so that it is equal to 72°/2 = 36°.
Since ∢ KGH = 144°/2 = 72°, and since triangle KGM is a right triangle, then ∢ GKM must be complementary to 72°, which is 90° - 72° = 18°.
∢ FKM = ∢ FKG + ∢ GKM = 36° + 18° = 54°. ∢ FKM + ∢ MKJ = 108°,
so that ∢ MKJ = 108° - 54° = 54°.
∢ HFK subtends arc HJK and thus ∢ HFK = 72°.
∢ GFH subtends arc GH and thus ∢ GFH = 36°.
Triangle FGH is isosceles, base ∢s are equal so that ∢ GHF = 36° and its complementary
∢ MPH = 90° - 36° = 54°.
[There are other methods for deriving these angles. Please note that every side has a parallel diagonal, e.g. KJ and FH].
2- Every ∢ including the arc ∢s has an SDQ = 9.
3- KM bisects GH so that GM = 1/2. In right triangle KGM, GM/KG = sin18°, so that KG = 1/(2sin18°) = Φ.
FH = KG, and every diagonal in the pentagon is equal to the Golden Proportion, Φ.
4- Draw diagonal GJ, (not shown), it will intersect FH at P, and form isosceles triangle PFG, where FG = FP = 1. Since FH = Φ, PH = FH - FP = $\Phi - 1 = 1/\Phi$, and FP/PH = Φ, again the Golden Proportion.