Solution 172

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The radius of any inscribed circle, also known as the incircle, is given by

$r = \sqrt{\dfrac{(s-a)(s-b)(s-c)}{s}}$, where s is the semiperimeter: $s = \dfrac{a+b+c}{2}$

We have b, c and r in this problem.

$s = ½(a + 528.390 + 622.118) = 575.254 + ½a$

$r^2 (575.254+½a)=(575.254–½a)(575.254+½a–528.39)(575.254+½a–622.118)$

$147.288^2(575.254)+10{,}846.8775\ a = (575.254–½a)(46.864+½a)(½a–46.864)$

which reduces to: $a^3 - 1150.508\ a^2 + 77990.082\ a + 109{,}946{,}496 = 0$

Any cubic equation of the form $ax^3 + bx^2 + cx + d = 0$ can be reduced to the form $y^3 + 3py + q = 0$ by letting $y = x + p/3a$

so that $p = [1/(9a^2)](3ac - b^2)$ and $q = (1/27)(2\ b^3 - 9abc + 27a^2d)$

(This solution does not require the coefficient of a^3 to be unity.)

$$p = \frac{(3)(1)(77{,}990.082) - (-1.150.508)^2}{(9)(1)} = -121{,}077.6013$$

$$q = \frac{(2)(-1{,}150.508)^3 - (9)(-1{,}150.508)(77{,}990.082) + 27(109{,}942{,}496)}{27}$$

$=27{,}045{,}131.53$

Letting $r = \sqrt{(-p^3)}$ = 42,130,412.63 and $\Phi = \cos^{-1}(-q/2r)$ = 108°43'17.578"

the solutions to the cubic equation in y are $y = 2\sqrt[3]{r}\cos\left(\dfrac{\Phi}{3}\right)$,

$y = 2\sqrt[3]{r}\cos\left(\dfrac{\Phi + 360°}{3}\right)$ and $y = 2\sqrt[3]{r}\cos\left(\dfrac{\Phi + 720°}{3}\right)$

y = 561.2924, –636.941 and 75.6486,

but, $y = x + p/3a$, or $x = y + 383.5027$

x = 944.795, –253.438 and <u>459.1513</u>, the latter being the solution for this problem