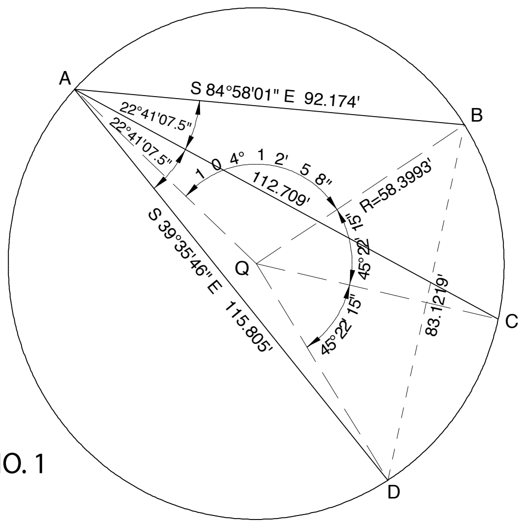
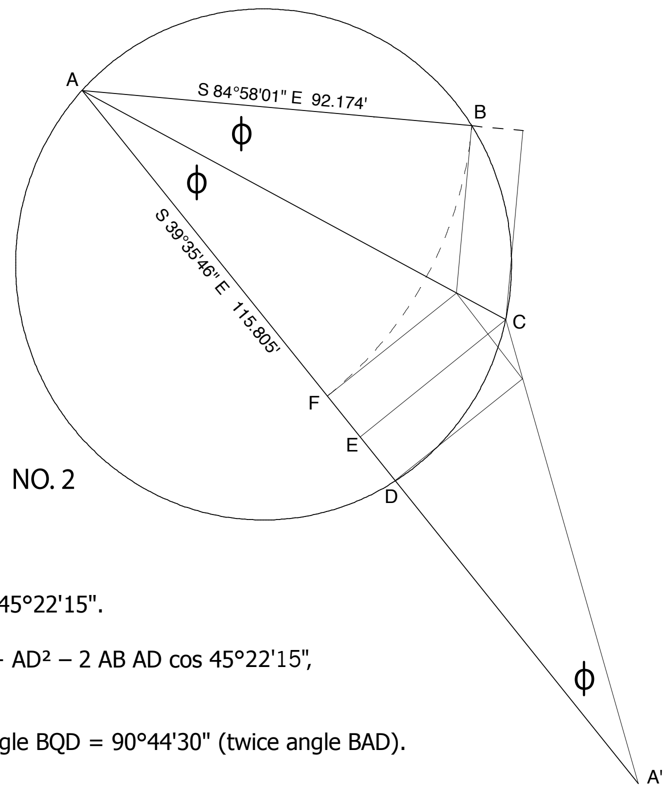
Solution
168

by Dave Lindell, L.S.



NO. 1



NO. 2

By difference of bearings, angle BAD = 45°22'15".

Using the Law of Cosines, $BD^2 = AB^2 + AD^2 - 2\ AB\ AD \cos 45°22'15"$,
so BD = 83.1219

In triangle BQD, BQ = QD = radius. Angle BQD = 90°44'30" (twice angle BAD).

Chord BD =2 R sin 45°22'15", so that R = 58.3993'

In triangle AQB by the Law of Cosines, $92.174^2 = R^2 + R^2 - 2 R^2 \cos$ angle AQB,
so that angle AQB = 104°12'58".

Angle AQB + angle BQC = 104°12'58" + 45°22'15" = 149°35'13"

Chord AC = 2(58.3993) sin[½(149°35'13")] = 112.709'

(Note: This is a weak solution, using cosines of angles between 120° & 180°.)

Alternatively,

Extend line AD to intersect CA', where CA' = AC. Construct AF = AB.

AA' = 2 AC cosϕ = AD + DA', so that 2AC cos ϕ – AD = DA'

AE – ½ FD = AF, and FD = AD – AF

2 AE – FD = 2 AF, and 2AE – (AD – AF) = 2 AF, and 2 AE – AD = AF

but AE = 2 AC cos ϕ, so DA' = AF

Therefore, 2 AC cos ϕ = AD + AF, but AF = AB

so, 2 AC cos ϕ = AD + AB, or AC = (AD + AB)/ 2 cos ϕ

(This is known as the Three Chord Lemma)

Solution
169

by Benjamin Bloch, Ph.D.

Solution to Problem 169

a) Here x = 2, and n = 15. (15-2)/6 = 2 **R** 1, so that SDQ(2) = 2, and **R = 1.** From the table the answer is ***8***. Thus, SDQ(2^{15}) = ***8***, and since SDQ(32,868) = 27=> ***9*** this answer must be ***incorrect***.
Correct answer: (32,768) => 26 => ***8***.

b) 16^7 =? 268,335,456.
SDQ(16) = **7**. For n = 7 we need not use the remainder calculation. From the table the answer is ***7***.
SDQ(268,335,456) = 42 => ***6***. So this answer is ***incorrect***.
Correct answer: (268,435,456) => 43 => ***7***.

c) 8^{23} =? 590,295,810,358,305,651,712 => 85 => 13 => ***4***.
(23 - 2)/6 = 3 **R = 3**
From the table we get ***8***. So this answer is ***incorrect***.
Correct answer: (590,295,810,358,705,651,712) => 89 => 17 => ***8***.

d) 147^{28} =? 4,840,445,926,998,527,143,180,132,566,802,461,408,607,116,960,093,883,732,904,561=> 263 => 11 => ***2***
SDQ(147) = 12 => **3**
(28-2)/6 = 4 R **2**
From the table the answer is ***9***. Therefore the answer given is ***incorrect***.
Correct answer: (4,840,445,926,998,527,143,180,132,566,802,461,408,607,116,960,091,883,732,904,561) => 261 => ***9***.