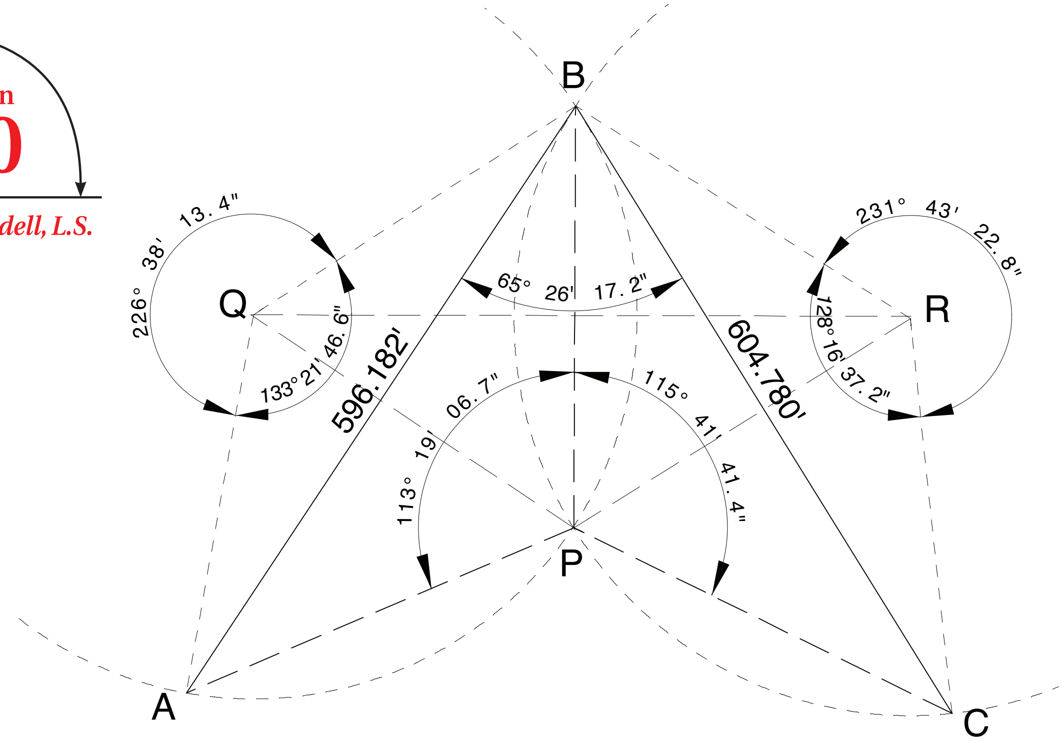
Solution 160

by Dave Lindell, L.S.



Angle AQB (the one greater than 180°) = 2(113°19′06.7″) = 226°38′13.4″
making angle AQB (the one less than 180°) = 133°21′46.6″
Therefore, angle QAB = QBA = ½ (180° – 133°21′46.6″) = 23°19′06.7″
AB = 596.182 = AQcos23°19′06.7″ + BQcos23°19′06.7″, but AQ = AB,
so, 2AQcos23°19′06.7″ = 596.182 and AQ = QB = 324.6053

Also, angle BRC (the one greater than 180°) = 2(115°51′41.4″) = 231°43′22.8″ and
angle BRC (the one less than 180°) = 128°16′37.2″
angle RBC = angle RCB = ½ (180° – 128°16′37.2″) = 25□ °51′41.4″
CB = 604.780 = CRcos25°51′41.4″ + BRcos25°51′41.4″, but CR = BR
so, 2CRcos25°51′41.4″ = 604.780 and CR = BR = 336.0445

$QR^2 = QB^2 + BR^2 – 2\,QB\,BR\cos(23°19′06.7″ + 65°26′17.2″ + 25°51′41.4″)$
$= 324.6053^2 + 336.0445^2 – (2)(324.6053)(336.0445)\cos 114°37′05.3″$
QR = 556.0347

$$\frac{\sin BRQ}{324.6053} = \frac{\sin BQR}{336.0445} = \frac{\sin 114°37′05.3″}{556.0347}$$

angle BRQ = 32°03′15.4″, angle BQR = 33°19′39.3″

(Note: 114°37′05.3″ + 32°03′15.4″ + 33°19′39.3″ = 180°00′00.0″)

BP = 2(324.6053)(sin 33°19′39.3″) = 356.6926
[as a check, BP = 2(336.0445)(sin 32°03′15.4″) = 356.6927]

Because QR is perpendicular to BP, angle BRQ=angle QRP and angle BQR=angle PQR
angle BPR = 90° – 32°03′15.4″ = 57°56′44.6″
angle BPQ = 90° – 33°19′39.3″ = 56°40′20.7″

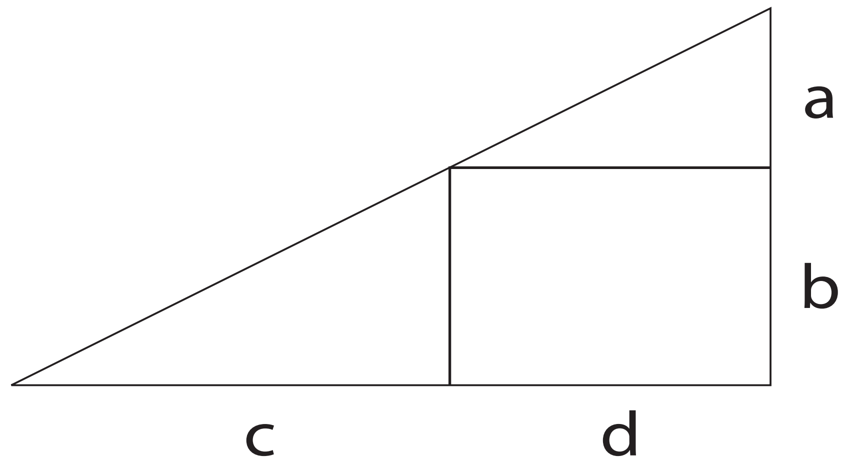
angle QPA = angle QAP = 113°19′06.7″ – 56°40′20.7″ = 56°38′46.0″
angle RPC = angle PRC = 115°51′41.4″ – 57°56′44.6″ = 57°54′56.8″

AP = (2)(324.6053)[sin½[180° – (2)(56°38′46.0″)] = 356.9416

PC = (2)(336.0445)[sin½[180° – (2)(57°54′56.8″)] = 356.9903

Solution 161

by Benjamin Bloch, Ph.D.



1. The total area of the inner triangles and rectangle must equal the area of the largest triangle. Thus, $1/2bc + 1/2ad + bd = 1/2(a + b)(c + d)$. Multiplying both sides by 2 yields: $bc + ad + 2bd = (a + b)(c + d) = ac + ad + bc + bd$, so that $2bd = ac + bd$, and finally $bd = ac$ the required relationship.
2. In this case, $bd = 3x5 = 15$ and $ac = 2x8 = 16$ and the answer is no. (See PSM Feb. 2008 problem #155.)
3. If $a = b$, then $d = c$.
4. The product ac must still equal bd, so that for a to equal c, a must also equal b.