Solution to Problem 156

by Dave Lindell, L.S.

Angle MQS =arcsin (400/960.81) = 24°36′08″, which is also the radial bearing to point S (the two radii lie on a North line).

Angle HOT = arcsin (400/694.88) = 37°38′50″, the radial bearing to point T.

OJ = 687.88, LQ = 927.81, LO = 687.88 – 353.53 = 334.35, and OQ = 927.81 – 334.35 = 593.46

In triangle OQP, by the Law of Cosines,

$$\text{cosine angle QOP} = \frac{(OQ)^2 + (OP)^2 - (QP)^2}{2(OQ)(OP)} = -0.182805953$$

and angle QOP = 100°32′00″, making angle POJ = 79°28′00″ and angle PON = 10°32′00″

PN = 654.88 cos 10°32′00″ = 643.8444

Angle QPN = arcsin (643.844/960.81) = 42°04′31″, the radial to point P

Arc SP = (42°04′31″ – 24°36′08″)(π/180)(960.81) = 293.01′

Arc PT = (79°28′00″ – 37°38′50″)(π/180)(654.88) = 477.99′

OH = 654.88 cos 37°38′50″ = 518.525

QM = 960.81 cos 24°36′08″ = 873.588

ST = OQ + OH – OM = 238.397

Area = area triangle POQ + area sector POK – area sector PQS

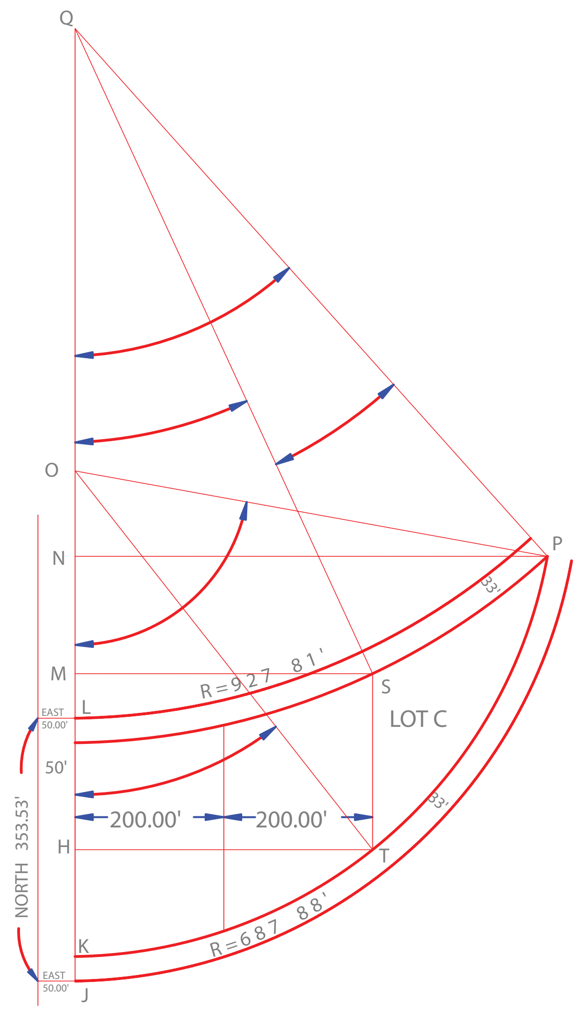
– area triangle SOQ – area triangle SOT – area sector TOK

= ½(593.46)(643.844) + ½(654.88^2)(79°28′00″)(π/180)

– ½(960.81^2)(17°28′23″)(π/180) – ½(593.46)(400) –

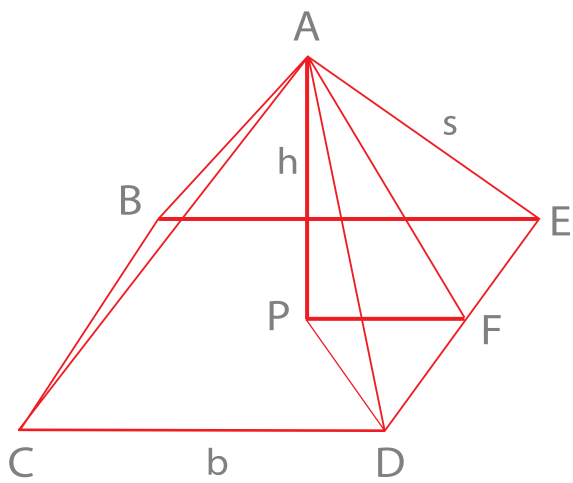
½(238.40)(400) – ½(654.88^2)(37°38′50″)(π/180)

= 191,047.830 + 297,410.175 – 140,763.890 – 118,692.000 – 47,680.000 – 140,897.655 = 40,424.46 square feet



Solution to Problem 157

by Benjamin Bloch, Ph.D.



Draw point F that bisects base side DE.

Draw lines PD and PF. PF = DF = FE = b/2.

The perimeter of the square base is 4b. The circumference of the circle, whose radius is height h, is $2\pi h$.

Thus, according to the design condition $4b = 2\pi h$, so that $PF = b/2 = \pi h/4$.

1- The Pyramid angle AFP = arc tan $h/(\pi h/4)$ = arc tan $4/\pi = 51.85°$ degrees´

2- To find the area of a side ADE in terms of h, we write PF as $b/2 = \pi h/4$, and from right triangle APF we find that $AF = (h/4)\,(16 + \pi^2)^{1/2}$ and the face triangle ADE has an area of $1/2\ AF \times DE = h^2\,(\pi/16)\,(16 + \pi^2)^{1/2} \approx h^2$. To recap, each face area is approximately equal to the square of the pyramid height.

3- In right triangle AFE, hypotenuse s equals the square root of $(AF)^2 + (FE)^2$ which equals $h\,(1 + \pi^2/8)^{1/2} \approx 1.4945568 \approx 3h/2$.