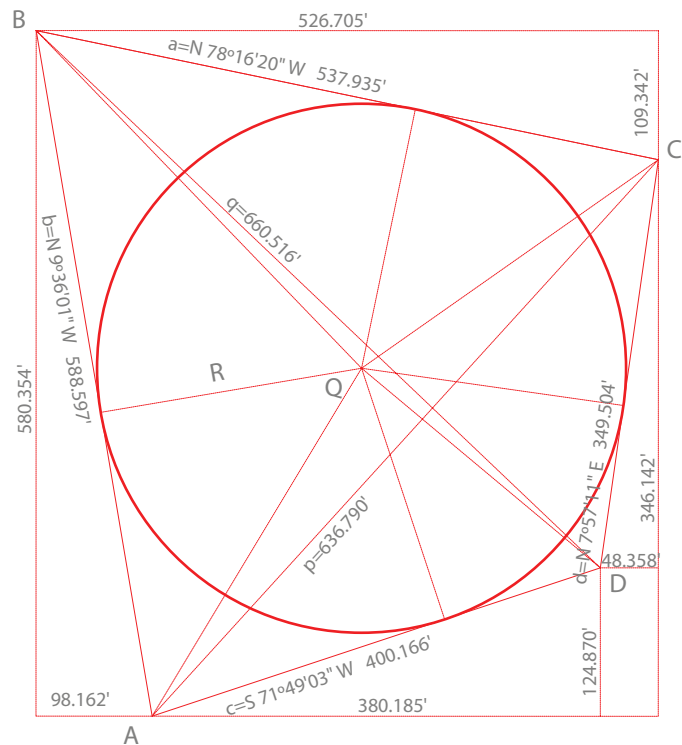


problem corner solution

Solution to
Problem
150

by Dave Lindell, L.S.



Area A-Q-B = $\frac{1}{2} (R)(AB)$, Area B-Q-C = $\frac{1}{2} (R)(BC)$,
Area C-Q-D = $\frac{1}{2} (R)(CD)$, and Area A-Q-D = $\frac{1}{2} (R)(AD)$
Total area = $\frac{1}{2} (R)(AB + BC + CD + DA) = \frac{1}{2}$ perimeter $\times R$

Using triangles and squares the total area is
 $580.354 \times 526.705 = 305,675.35$ sq. ft.
 $-\frac{1}{2} \times 580.354 \times 98.162 = -28,484.35$ sq. ft.
 $-\frac{1}{2} \times 526.705 \times 109.342 = -28,795.49$ sq. ft.
 $-\frac{1}{2} \times 346.142 \times 48.358 = -8,369.37$ sq. ft.
 $-48.358 \times 124.870 = -6,038.46$ sq. ft.
 $-\frac{1}{2} \times 380.185 \times 124.870 = -23,736.85$ sq. ft.
 $= 210,250.83$ square feet

$210,250.83 = \frac{1}{2} (588.597 + 537.935 + 349.504 + 400.166) \times R$,
so that $R = 224.124$

(There is also a solution using tangents of half the angles subtended at Q.
The reader is left to find this method and solve it.)

Note: if $s = \frac{1}{2} (a + b + c + d)$, where a, b, c, and d are the sides

$$R = \frac{\sqrt{abcd}}{s} = \frac{\sqrt{588.597 \times 537.935 \times 349.504 \times 400.166}}{\frac{1}{2} (588.597 + 537.935 + 349.504 + 400.166)}$$

$$R = \frac{210,436.026}{938.101} = 224.321' \text{ Why doesn't this check?}$$

The square root solution only works if the quadrilateral is concyclic, that is, if it can be circumscribed.

Use Ptolemy's Theorem to verify:

$a \times c + b \times d = p \times q$, where a, b, c, and d are the sides and p and q the diagonals

$$537.935 \times 400.166 + 588.597 \times 349.504 = 420,980.303$$

$$636.790 \times 660.516 = 420,609.984$$

Since the equality fails, the quadrilateral is not concyclic and cannot be circumscribed.

The only condition for an inscribed circle is that $a + c = b + d$



Solution to
Problem
151

by Benjamin Bloch, Ph.D.

Let R = radius at the surface of the Earth, r = radius at depth, and d = depth. For concentric spheres, the ratio of the projected area to the surface area equals the ratio of the total projected area to the total surface area.

Now the total surface area of a sphere of radius r is $4\pi r^2$. Thus, the total area of the sphere below is $4\pi r^2$ while the total area on the surface of the Earth is $4\pi R^2$. The ratio of these areas (below to above) is $\frac{1}{2}$ acre to 1 acre so that $(4\pi r^2/4\pi R^2) = (\frac{1}{2} / 1)$ and putting in the value of $R = 3,963$ miles we can solve for r , the radius at which the projected area is $\frac{1}{2}$ acre. We find that $r = R/\sqrt{2}$ and thus, the depth, $d = R - r = R - R/\sqrt{2} = 1,161$ miles (SDQ = 9).