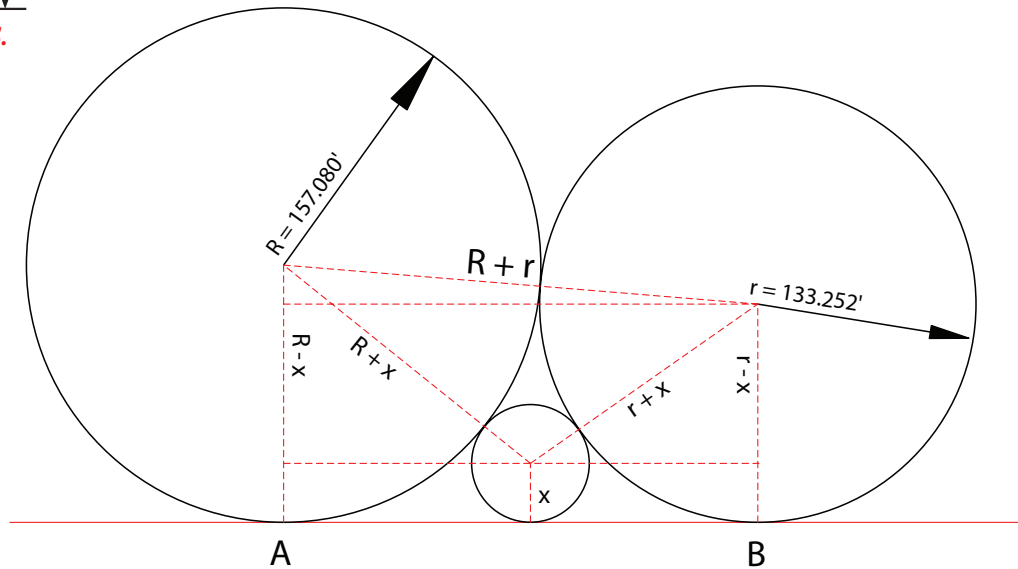


# problem corner solution

Solution to  
Problem  
**142**

by Dave Lindell, L.S.



$$\begin{aligned}(AB)^2 + (R - r)^2 &= (R + r)^2 \\(AB)^2 &= R^2 + 2Rr + r^2 - R^2 + 2Rr - r^2 \\ &= 4Rr\end{aligned}$$

$$AB = 2\sqrt{Rr}$$

$$\sqrt{[(R + x)^2 - (R - x)^2]} + \sqrt{[(r + x)^2 - (r - x)^2]} = AB = 2\sqrt{Rr}$$

$$\sqrt{4Rx} + \sqrt{4rx} = 2\sqrt{Rr}$$

$$\sqrt{Rx} + \sqrt{rx} = \sqrt{Rr}$$

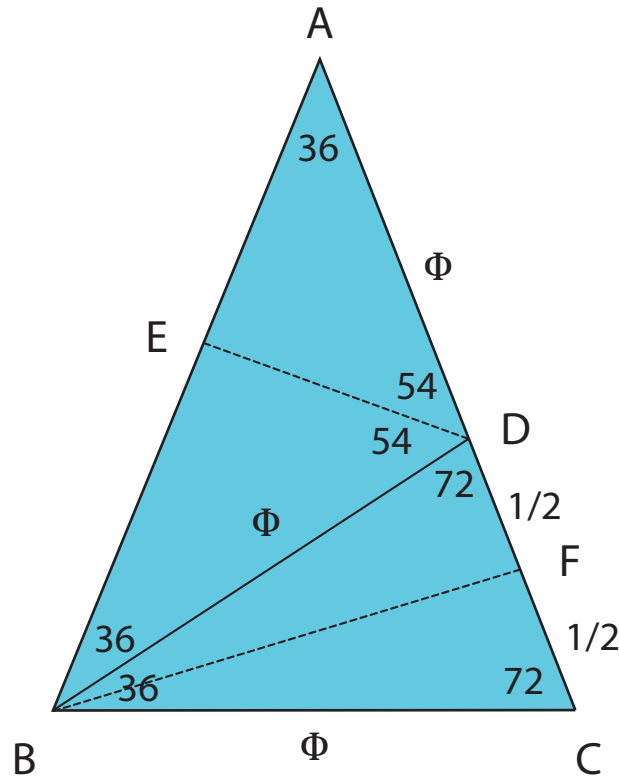
$$\sqrt{x} (\sqrt{R} + \sqrt{r}) = \sqrt{Rr}$$

$$x = \frac{Rr}{(\sqrt{R} + \sqrt{r})^2} = \frac{(157.080)(133.252)}{(12.53315603 + 11.54348301)^2} = 36.108$$

$$\text{Note: } \frac{1}{\sqrt{x}} = \frac{\sqrt{R} + \sqrt{r}}{\sqrt{Rr}} = \frac{1}{\sqrt{r}} + \frac{1}{\sqrt{R}}$$

Solution to  
Problem  
**143**

by Benjamin Bloch, Ph.D.



- Adding the angles in triangle ABC we get  $5a = 180$ , so that  $a = 36$ .  
ADE and BDE are Right Triangles, thus,  $b = c = 90 - a = 90 - 36 = 54$  and  $d = 180 - 2b = 72$ .  
Thus,  $a = 36$ ;  $b = c = 54$ , and  $d = 72$ .
- Draw BF perpendicular to DC, forming congruent triangles DBF and CBF, thus bisecting DC.  
Now  $BC = \frac{1}{2} \sec 2a = \frac{1}{2} \sec 72 = 1.618$ . Hence  $BC = BD = AD = 1.618$ .  
 $AC = AB = 2.618$ .  $AE = AB/2 = 1.309$ .  
Thus,  $BC = AD = 1.618$ ,  $AE = 1.309$ , and  $AB = 2.618$ .
- Every angle including CBF and DBF has a SDQ of 9.
- BC whose value is 1.618, rounded to 1.62 (SDQ of 9) is known as phi,  $\Phi$  the Golden Proportion, and triangle ABC is a Golden Triangle.