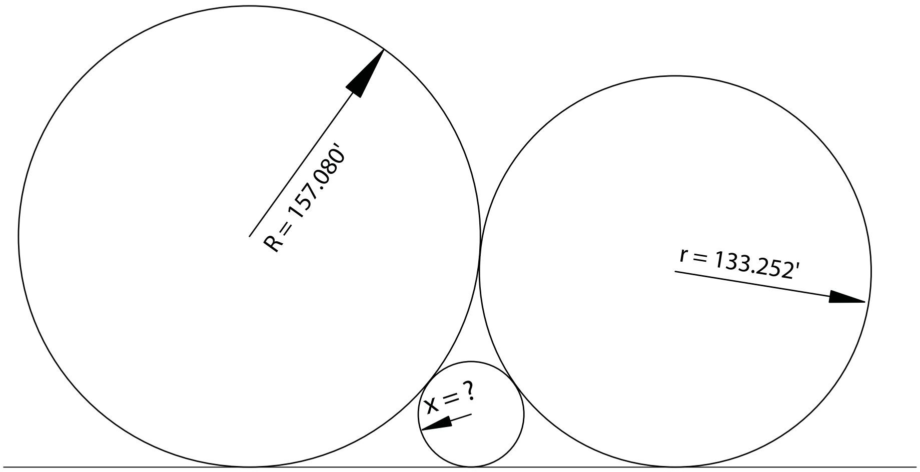
Problem 142

by Dave Lindell, L.S.



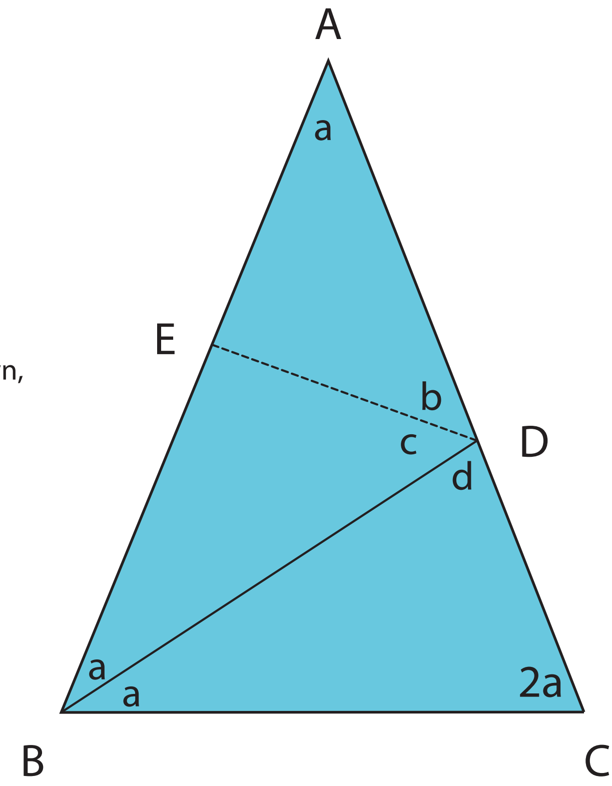
The two larger circles are tangent to a line and each other. The small circle is tangent to the two larger circles and the line. What is the radius of the small circle?

Problem 143

by Benjamin Bloch, Ph.D.

Given triangle ABC with angles a, b, c, d as shown, DE is perpendicular to AB, DC = 1.

Determine:
a) The numerical value of every angle.
b) The numerical values of: BC, AD, AE and AB.
c) What all the angles have in common.
d) Find phi, Φ, the Golden Proportion, in BC.

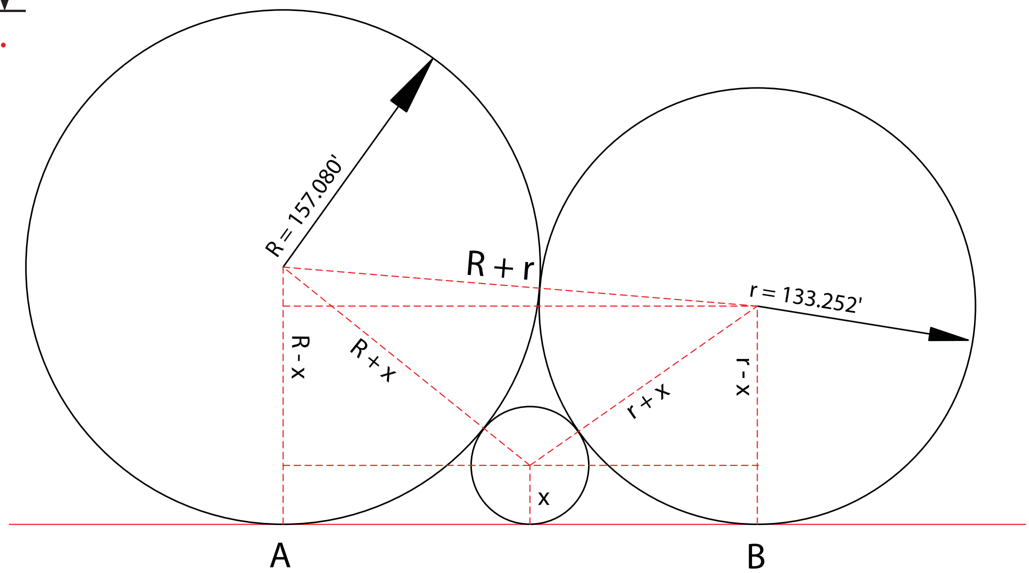


Solutions are on our website at *www.profsurv.com*

problem corner solution

Solution to Problem 142

by Dave Lindell, L.S.



$$(AB)^2 + (R - r)^2 = (R + r)^2$$

$$(AB)^2 = R^2 + 2Rr + r^2 - R^2 + 2\,Rr - r^2$$

$$= 4\,R\,r$$

$$AB = 2\sqrt{(R\,r)}$$

$$\sqrt{[(R + x)^2 - (R - x)^2]} + \sqrt{[(r + x)^2 - (r - x)^2]} = AB = 2\sqrt{(R\,r)}$$

$$\sqrt{4\,R\,x} + \sqrt{4\,r\,x} = 2\sqrt{R\,r}$$

$$\sqrt{Rx} + \sqrt{r\,x} = \sqrt{R\,r}$$

$$\sqrt{x}\,(\sqrt{R} + \sqrt{r}) = \sqrt{R\,r}$$

$$x = \frac{R\,r}{(\sqrt{R} + \sqrt{r})^2} = \frac{(157.080)(133.252)}{(12.53315603 + 11.54348301)^2} = 36.108$$

Note: $$\frac{1}{\sqrt{x}} = \frac{\sqrt{R} + \sqrt{r}}{\sqrt{(R\,r)}} = \frac{1}{\sqrt{r}} + \frac{1}{\sqrt{R}}$$

Solution to Problem 143

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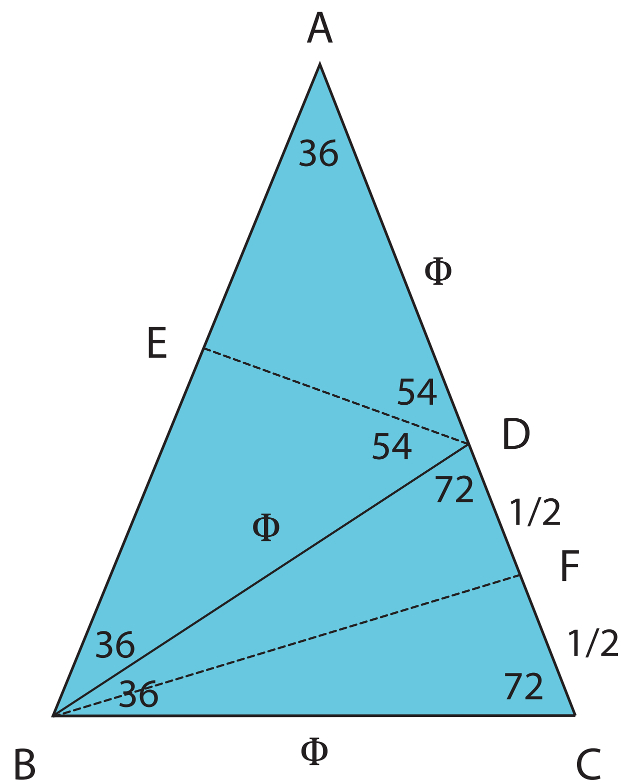

a) Adding the angles in triangle ABC we get 5a = 180, so that a = 36.
ADE and BDE are Right Triangles, thus, b = c = 90 – a = 90 – 36 = 54 and d = 180 – 2b = 72.
Thus, a = 36; b = c = 54, and d = 72.

b) Draw BF perpendicular to DC, forming congruent triangles DBF and CBF, thus bisecting DC.
Now BC = 1/2 sec 2a = ½ sec 72 = 1.618. Hence BC = BD = AD = 1.618.
AC = AB = 2.618. AE = AB/2 = 1.309.
Thus, BC = AD = 1.618, AE = 1.309, and AB = 2.618.

c) Every angle including CBF and DBF has a SDQ of 9.

d) BC whose value is 1.618, rounded to 1.62 (SDQ of 9) is known as phi, Φ the Golden Proportion, and triangle ABC is a Golden Triangle.