

problem corner solution

Solution to
Problem
137

by Benjamin Bloch, Ph.D.

The smallest integer right triangle is a 9, 12, 15 right triangle whose Perimeter = $9+12+15 = 36 \Rightarrow 9$, and whose Area = $1/2 (9 \times 12) = 54 \Rightarrow 9$.

We note that $AX = AZ$, and $BX = BY$, and $CZ = CY$ so that we have three unknowns.

Since $AX + XB = 15$, and $BY + YC = 12$, and $AZ + ZC = 9$, we have three equations with three unknowns.

Solving we get that $AX = AZ = 6$, $BX = BY = 9$, and $CZ = CY = 3$.

A) $AX = 6$, $XB = 9$, $BY = 9$, $YC = 3$, $AZ = 6$, and $ZC = 3$.

B) The length of the side $ZC = 3$, is the radius of the inscribed circle. The circle area $A = 9\pi$.

C) Each of the four triangles has the same height, 3 which is the radius of the circle. Triangles AZO and AXO are congruent and have areas of 9 each. Triangles BOX and BOY are congruent and have areas of 13.5 $\Rightarrow 9$ each. Since the area of the square is also 9, the total area of the four interior triangles plus the square is $54 \Rightarrow 9$.

D) Angle OBX equals $\arctan 1/3 = 18$ degrees (note that $18 \Rightarrow 9$) and angle OAZ equals $\arctan 1/2 = 27$ degrees (note that $27 \Rightarrow 9$)