problem corner solution

Solution to Problem 136

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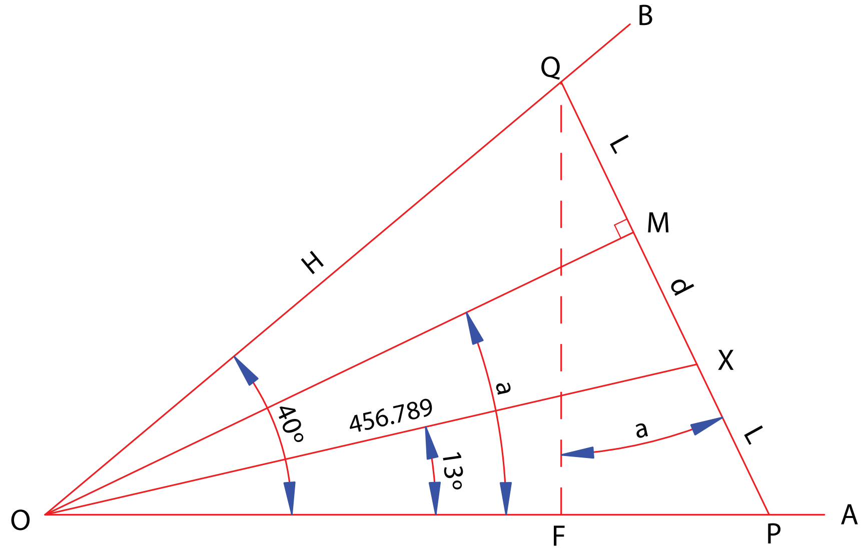

PQ is a minimum when QM = XP and OM is perpendicular to PQ.
Draw QF perpendicular to OA, forming angle FQP = angle AOM = a

$OM^2 = H^2 - L^2$

$\tan a = (d + L) / (\sqrt{H^2 - L^2})$

$\tan (a - 13°) = d / (\sqrt{H^2 - L^2}) = (\tan a - \tan 13°) / (1 + \tan a \times \tan 13°)$

$\tan (40° - a) = L / (\sqrt{H^2 - L^2}) = (\tan 40° - \tan a) / (1 + \tan 40° \times \tan a)$

$\tan (a - 13°) + \tan (40° - a) = (d + L) / (\sqrt{(H^2 - L^2)}) = \tan a$

$(\tan a - \tan 13°)/(1+\tan a \times \tan 13°) + (\tan 40° - \tan a)/(1+\tan 40° \times \tan a) = \tan a$

Letting tan a = m,

$[(m - \tan 13°)(1 + m \tan 40°)+(\tan 40° - m)(1 + m \tan 13°)] /$
$[(1 + m \tan 13°)(1 + m \tan 40°)]=m$

$[(m- 0.230868191)(1+0.839099631\ m)+(0.839099631-m)(1+0.230868191\ m)]/$
$[(1 + 0.2308689191\ m)(1+ 0.839099631\ m)] = m$

which, after expanding and rearranging becomes

$(0.60823144\ m^2 + 0.60823144) / (1+1.069967822\ m + 0.193721414\ m^2) = m$

and reduces to $m^3 + 2.383507184\ m^2 + 5.162051935\ m - 3.139722282$

From the *CRC Standard Mathematical Tables and Formulae* (29th Edition, pg. 9):
A cubic equation of the form $y^3 + py^2 + qy + r = 0$ can be reduced to the form $x^3 + ax + b = 0$ by letting $y = x - p/3$ so that

$a = 1/3\ (3q - p^2)$ and

$b = 1/27(2p^3 - 9pq + 27r)$

Letting A = the cube root of $[(-b/2 + \sqrt{(b^2/4 + a^3/27)}]$

and B = the cube root of $[-b/2 - \sqrt{(b^2/4 + a^3/27)}]$,

x = A + B and two other complex roots

For this problem, p= 2.383507184, q= 5.162051935 and r= −3.139722282

a = 1/3(15.48615581 − 5.681106496) = 3.26834977

b = 1/27(27.0819163 − 110.7340908 − 84.77250161 = − 6.237950968

A = 1.860377645

B = − 0.585606866

So that x = 1.274770779

m = 1.274770779 − 2.383507184/3 = 0.480268383 = tan a

a = 25°39'12.6"

L = 456.789 sin 13° / cos a = 113.9915

d = 456.789 sin (a − 13°) = 100.0617

Line PQ = 328.045