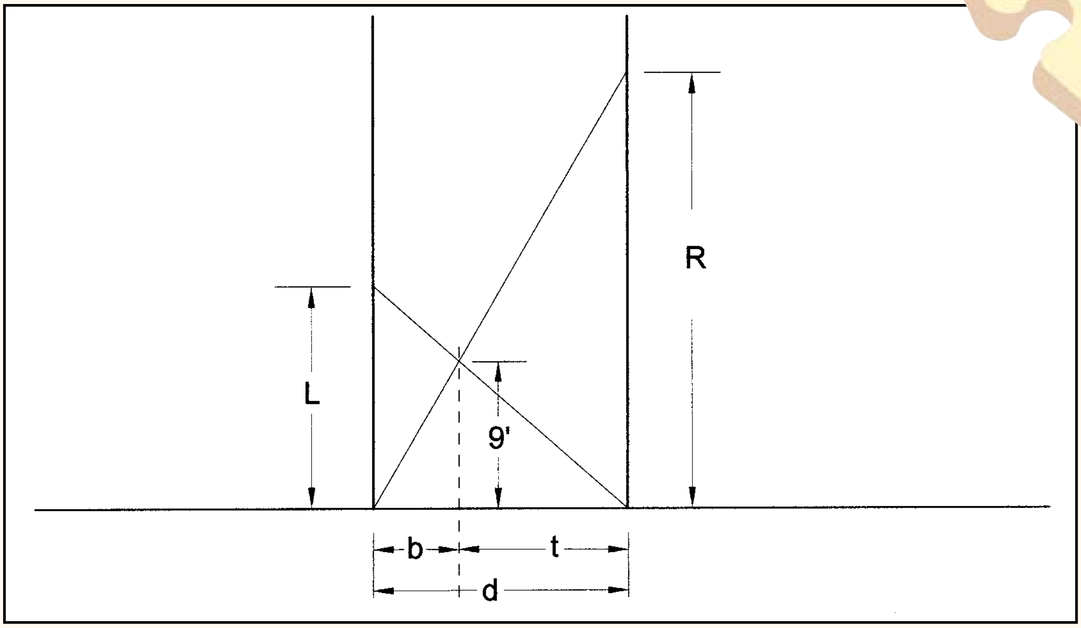
Solution to Problem 117



By similar triangles, $\dfrac{b}{9} = \dfrac{d}{R}$ and $\dfrac{t}{9} = \dfrac{d}{L}$

$b = 9d / R$ and $t = 9d / L$

$9d / R + 9d / L = d$

$1 / R + 1 / L = 1 / 9$

$L^2 = 20^2 - d^2 = 400 - d^2$, so that $d^2 = 400 - L^2$

$R^2 = 30^2 - d^2 = 900 - d^2 = 900 - 400 + L^2 = 500 + L^2$

$1 / \sqrt{(500 + L^2)} + 1/ L - 1/9 = 0$ (1)

There is no direct solution, so iteration is necessary:

For L = 5, (1) yields 0.0933

For L = 10, (1) yields 0.0297 (The trend is towards zero)

For L = 15, (1) yields –0.0073 (Too far)

Linear interpolation between 10 and 15 gives L=14.0135, then (1) is –0.00186

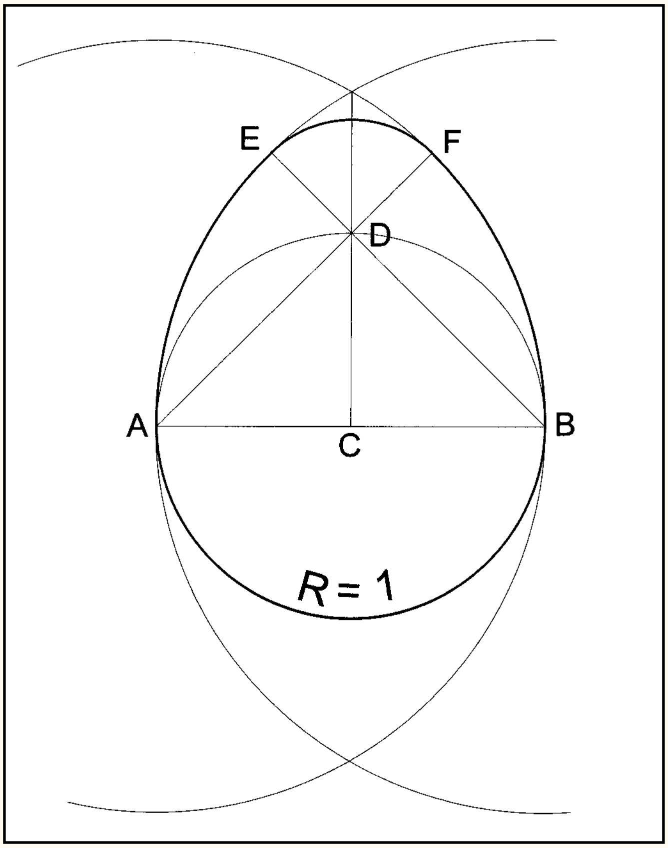
Returning to L = 13, (1) yields 0.0045

Interpolation between 13 and 14.0135 gives L= 13.7163 and (1) is 0.00008

$d^2 = 400 - L^2 = 400 - 13.7163^2 = 211.863$, $d = 14.556'$

Note: Equation (1) can be expanded into a fourth degree equation with one unknown and solved explicitly, but the algebra is tedious and complicated.

Solution to Problem 118



A-C = C-B = C-D = 1

A-D = D-B = $\sqrt{2}$

E-D = D-F = $2 - \sqrt{2}$

The area below line A-C-B, $A_1 = \frac{1}{2}\pi\, 1^2 = \pi / 2$

Sector A-B-E = sector F-A-B = $1/8\ \pi\ 2^2 = \pi / 2$

Sector E-D-F = $\frac{1}{4}\pi\,(2 - \sqrt{2})^2 = 0.085786438\,\pi$

Triangle A-D-B = $2 \times 1 \times 1 \times \frac{1}{2} = 1$

$\text{Area}_{egg} = A_1$ + sector A-E-B + sector F-A-B + sector E-D-F – Δ A-D-B

$= \pi/2 + \pi/2 + \pi/2 + 0.085786438\,\pi - 1 = 3.981895023$