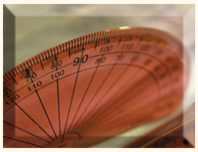
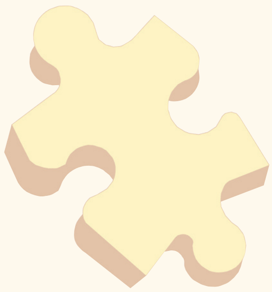
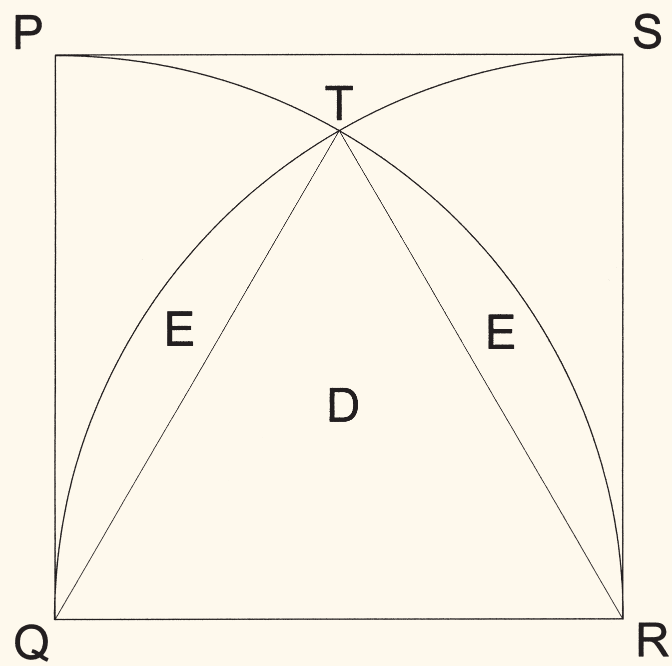


PROBLEMCORNER

Solution to Problem 113





Let square PQRS be *1 mile* on a side.

(1) Area A + 4 x area B + 4 x area C = 1 square mile.

(2) Area A + 3 x area B + 2 x area C = ¼ of a circle = $\pi/4$

(3) Area A + 2 x area B + area C = 2 x area E + area D

Triangle QTR is equilateral and has an area of $\sqrt{3} / 4$

2 area E + area D = Sector TQR + sector QRT – triangle QTR

Sector TQR = Sector QRT = $\pi/6$

(1) $A + 4B + 4C = 1$
(2) $A + 3B + 2C = \pi/4$
(3) $A + 2b + C = \pi/3 - \sqrt{3} / 4$

Combining (1) and (2), $B + 2C = 1 - \pi/4$ (4)

Combining (2) and (3), $B + C = \pi /4 - \pi/3 + \sqrt{3} / 4$ (5)

Combining (4) and (5), $C = 1 - \sqrt{3} / 4 - \pi/6 = 0.043388523$ sq. mi.

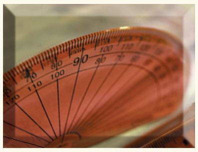
By substitution of C into (5), $B + 1 - \sqrt{3} / 4 - \pi/6 = \pi/4 - \pi/3 + \sqrt{3} / 4$

and $B = \pi/12 + \sqrt{3} /2 - 1 = 0.127824792$ sq. mi.

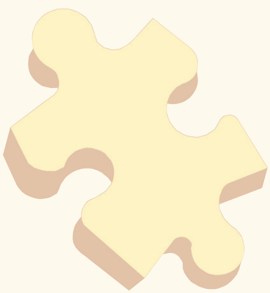
By substitution of B and C into (2),

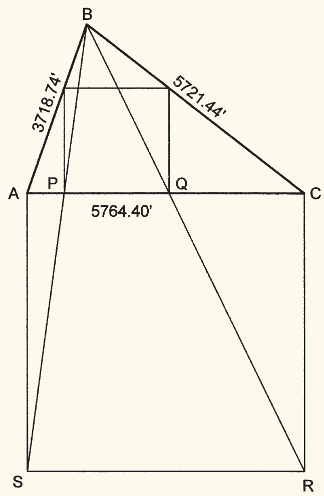
$A + 3(\pi/12 + \sqrt{3} / 2 - 1) + 2(1 - \sqrt{3} / 4 - \pi/6)$

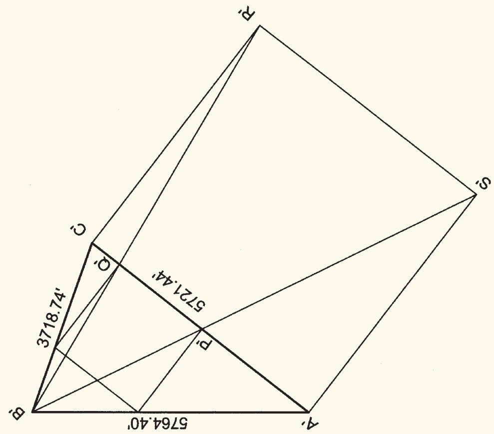
And $A = 1 + \pi/3 - \sqrt{3} = 0.315146744$ sq. mi.

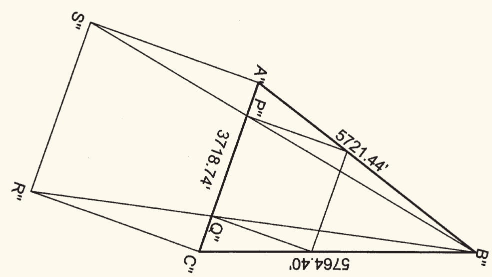


Solution to Problem 114









First, solve for the angles in the triangle by the Law of Cosines:

A = 70°29′02.4″, B = 71°44′11.5″, C = 37°46′46.1″

Construct square A-C-R-S on side AC. Construct lines BS and BR, intersecting AC at P and Q. Construct perpendiculars to AC at P and Q. The intersections with sides AB and BC are the corners of the square as are points P and Q.

Let AC bear East. Traverse from A to B (or C to B, or both for a check) using the complements of the angles calculated for the triangle as bearings. Traverse to S and R, or just assign coordinates to them. Inverse from S to B and from R to B.

SB = N 7°38′00″ E, RB = N 26°00′19 W (No distances are needed)

AP = 5764.40 x tan 7°38′00″ = 772.56′

CQ = 5764.40 x tan 26°00′19″ = 2812.14′

PQ = 5764.40 – 772.56 – 2812.14 = 2179.70′

Now, do the same for the other two sides: (Just substitute A' or A" for A, etc.)

The bearing for S'B' is N 26°12′55″ E and for R'B' is N 7°10′43″ W
A'P' = 2817.195′, C'Q' = 720.62′, making P'Q' = 2183.625′

The bearing for S"B" is N 11°05′05″ E and for R"B" is N 11°52′57″ W
A"P" = 728.565′, C"Q" = 782.48′, making P"Q" = 2207.695′

The square constructed on the shortest side is the largest.