Solution to Problem 109

The formula $A = P(1 + r/n)^{nt}$, where A is the amount, P is the principal, r is the rate, n is the number of compounding periods and t, the time period, is used to calculate compound interest.

In your case, P = 10,000,000, r= 100% = 1.00, n is the number you choose and t = 1, one year.

Compounded monthly, $A=\$10,000,000(1+1/12)^{12} = \$26,130,352.90$

Weekly, $A = 10,000,000(1+1/52)^{52} = 26,925,969.54$

Daily, $A = 10,000,000(1 + 1/365)^{365} = 27,145,674.82$

Hourly, $A = 10,000,000(1 + 1/8760)^{8760} = 27,181,266.92$

By the minute, $A = 10,000,000(1 + 1/525600)^{525600} = 27,182,792.43$

By the second, $A = 10,000,000(1 + 1/31536000)^{31536000} = 27,182,817.85$

For 100,000,000, $A = 10,000,000(1 + 1/100,000,000)^{100,000,000} = 27,182,818.15$

Recognize this number?

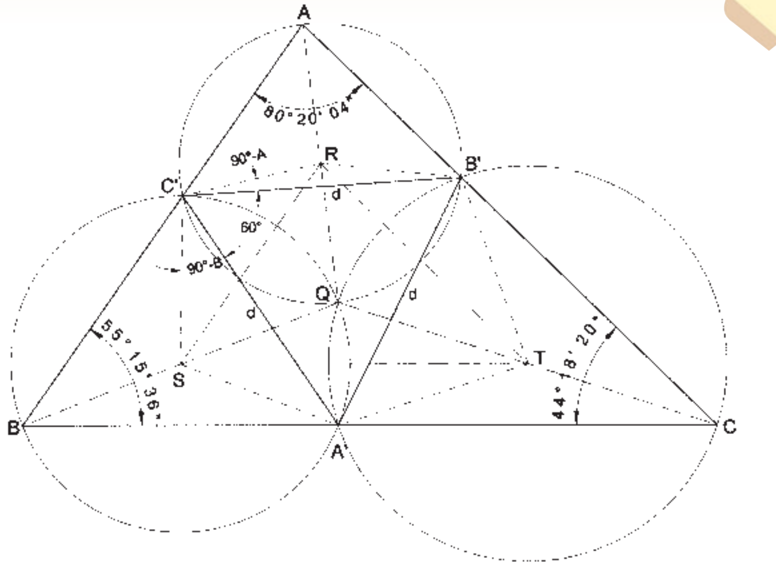
It is approaching 10,000,000e, where e is the base of the natural logarithms.

In fact, the limit of $(1 + w)^{1/w}$ as w approaches zero is the definition of e.

(e = 2.71828 18284 59045 23536 02874 71352 66249 77572 ...)

(The real answer to this problem is "zero". It was a dream, remember?)

Solution to Problem 110



Construct circles with centers at R, S and T that circumscribe triangles A-B'-C', B-C'-A' and C-A'-B' respectively. Because angle A-C'-Q is a right angle, line A-Q must be a diameter, as must B-Q and C-Q. Draw lines R-S, S-T and T-R. Line R-S is perpendicular to a line joining C'-Q, a chord of the circle with center at S or R, and therefore parallel with line A-B'. Side R-S can be shown by similar triangles to be one-half the length of side A-B, S-T half that of B-C and R-T half that of A-C.

From the given sides, calculate the angles of triangle A-B-C by the Law of Cosines: A = 80°20'04", B = 55°15'36", C = 44°18'20".

Angle C'-R-B' = twice angle A, and angle C'-S-A' = twice angle B

(C'-B') = 2 (C-'R) sin A, so that (C'-R) = (C'-B')/2 sin A = d/2 sin A
(C'-A') = 2 (C'-S) sin B, so (C'-S) = (C'-A')/2 sin B = d/2 sin B
where d is the side of the equilateral triangle

Angle R-C'-B' = angle R-B'-C' = 90° – angle A
Angle S-C'-A' = angle S-A'-C' = 90° – angle B
Angle R-C'-S = 90° – angle A + 90° – angle B + 60°
= 180° – angle A – angle B + 60°
but 180° – angle A – angle B = angle C
so angle R-C'-S = angle C + 60°

From the Law of Cosines, $(R\text{-}S)^2 = (C'\text{-}S)^2 + (C'\text{-}R)^2 - 2(C'\text{-}S)(C'\text{-}R)\cos(C+60°)$

$425^2 = d^2/4 \sin^2 B + d^2/4 \sin^2 A - (2)(d/2 \sin B)(d/2 \sin A)\cos(C+60°)$

Multiplying both sides by $4 \sin^2 A \sin^2 B$,

$425^2(4 \sin^2 A \sin^2 B) = d^2 \sin^2 A + d^2 \sin^2 B - 2d^2 \sin B \sin A \cos(C+60°)$

$d^2 = 425^2(4)(\sin^2 A \sin^2 B) / [\sin^2 A + \sin^2 B - 2 \sin B \sin A \cos(C+60°)]$

$d = 850 \sin A \sin B / \mathrm{sqrt}[\sin^2 A + \sin^2 B - 2 \sin B \sin A \cos(C + 60°)]$

with all values on the right side known, d= 481.283'