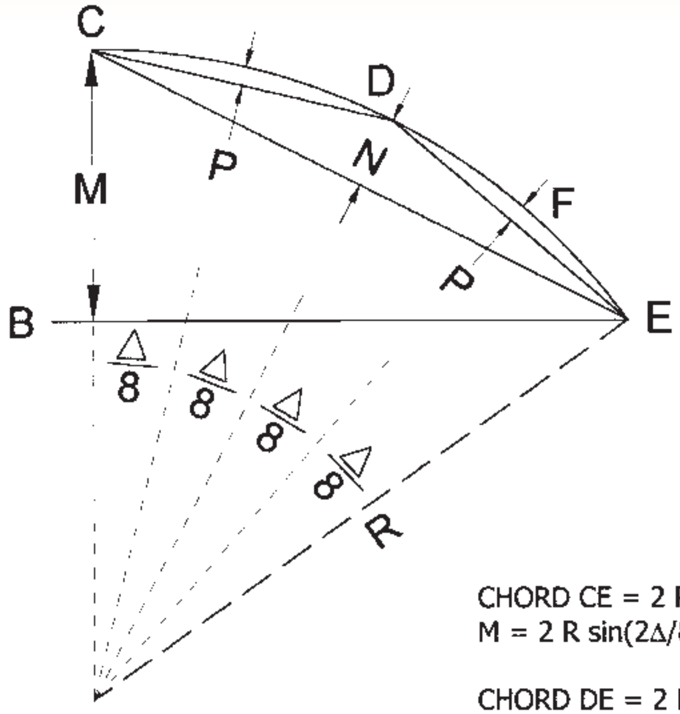
Solution to Problem 97



CHORD CE = 2 R sin(2Δ/8), ANGLE C-E-B = 2Δ/8
M = 2 R sin(2Δ/8) sin(2Δ/8)

CHORD DE = 2 R sin(Δ/8), ANGLE D-E-C = Δ/8
N = 2 R sin(Δ/8) sin(Δ/8)

CHORD FE (not shown) = 2R sin(Δ/16), ANGLE F-E-B =Δ/16
P = 2 R sin(Δ/16) sin(Δ/16

$$\frac{N}{M} = \frac{2\ R \sin(\Delta/8) \sin(\Delta/8)}{2\ R \sin\ (2\Delta/8) \sin\ (2\Delta/8)}\ ,\ \text{LET}\ \Delta/8 = \phi$$

$$= \frac{\sin\phi \sin\phi}{\sin 2\phi \sin 2\phi} = \frac{\sin\phi \sin\phi}{(2 \sin\phi \cos\phi)\ (2 \sin\phi \cos\phi)}$$

$$= \frac{1}{4 \cos^2 \phi} = \tfrac{1}{4} \sec^2\phi = \tfrac{1}{4} \sec^2(\Delta/8)$$

The same logic can be applied to P and N, which yields the same ratio.

The relation of P to M is $(1/16) \sec^2(\Delta/16) \sec^2(\Delta/8)$

Solution to Problem 98

For M to be an integral multiple of N, say Q,

$M = R - R\cos(4\Delta/8)$ must be Q times $N = R - R\cos(2\Delta/8)$

$R\,[1 - \cos(4\Delta/8)] = QR[1 - \cos(2\Delta/8)$

Letting $2\Delta/8 = \phi$, $1 - \cos 2\phi = Q\,(1 - \cos\phi)$

$[1 - (2\cos^2\phi - 1)] = Q - Q\cos\phi$

Expanding and rearranging, $2\cos^2\phi - Q\cos\phi + (Q-2) = 0$

Solving for cosφ by the quadratic equation, $\cos\phi = \dfrac{2Q - 4}{4}$ & -1

For Q=1, cosφ=– 0.5 & –1, φ=120°, 180°, 240° and Δ=720°>180°

For Q=2, cosφ=0 & –1, φ=90°, 180°, 270° and Δ=360°>180°

For Q=3, cosφ=0.5 & –1, φ=60°, 180°, 300° and Δ=240°>180°

For Q=4, cosφ=1.0 & –1, φ=0°, 180°, 360° and Δ=720°?180°

For Q > 5, cosφ > 1, not a real number solution.

There are no integral multiples of N that equal M for Δ<180°.