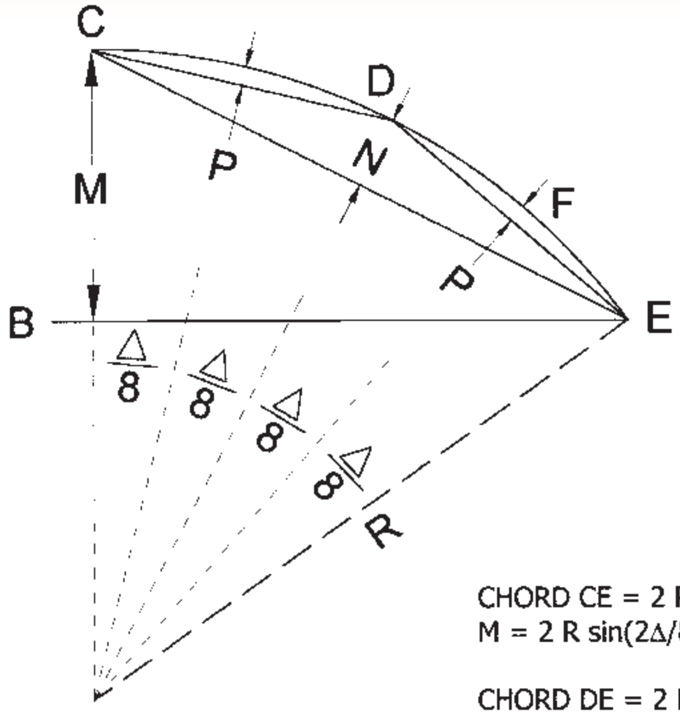
Solution to Problem 97



CHORD CE = $2\,R \sin(2\Delta/8)$, ANGLE C-E-B = $2\Delta/8$
M = $2\,R \sin(2\Delta/8)\sin(2\Delta/8)$

CHORD DE = $2\,R \sin(\Delta/8)$, ANGLE D-E-C = $\Delta/8$
N = $2\,R \sin(\Delta/8)\sin(\Delta/8)$

CHORD FE (not shown) = $2R \sin(\Delta/16)$, ANGLE F-E-B = $\Delta/16$
P = $2\,R \sin(\Delta/16)\sin(\Delta/16$

$$\frac{N}{M} = \frac{2\,R \sin(\Delta/8)\sin(\Delta/8)}{2\,R \sin(2\Delta/8)\sin(2\Delta/8)}, \quad \text{LET } \Delta/8 = \phi$$

$$= \frac{\sin\phi \sin\phi}{\sin 2\phi \sin 2\phi} = \frac{\sin\phi \sin\phi}{(2\sin\phi\cos\phi)(2\sin\phi\cos\phi)}$$

$$= \frac{1}{4\cos^2\phi} = \tfrac{1}{4}\sec^2\phi = \tfrac{1}{4}\sec^2(\Delta/8)$$

The same logic can be applied to P and N, which yields the same ratio.

The relation of P to M is $(1/16)\sec^2(\Delta/16)\sec^2(\Delta/8)$

Solution to Problem 98

For M to be an integral multiple of N, say Q,

$M = R - R\cos(4\Delta/8)$ must be Q times $N = R - R\cos(2\Delta/8)$

$R\,[1 - \cos(4\Delta/8)] = QR[1 - \cos(2\Delta/8)$

Letting $2\Delta/8 = \phi$, $1 - \cos 2\phi = Q\,(1 - \cos\phi)$

$[1 - (2\cos^2\phi - 1)] = Q - Q\cos\phi$

Expanding and rearranging, $2\cos^2\phi - Q\cos\phi + (Q-2) = 0$

Solving for cosϕ by the quadratic equation, $\cos\phi = \dfrac{2Q-4}{4}$ & -1

For Q=1, $\cos\phi = -0.5$ & -1, $\phi = 120°, 180°, 240°$ and $\Delta = 720° > 180°$

For Q=2, $\cos\phi = 0$ & -1, $\phi = 90°, 180°, 270°$ and $\Delta = 360° > 180°$

For Q=3, $\cos\phi = 0.5$ & -1, $\phi = 60°, 180°, 300°$ and $\Delta = 240° > 180°$

For Q=4, $\cos\phi = 1.0$ & -1, $\phi = 0°, 180°, 360°$ and $\Delta = 720°?180°$

For $Q > 5$, $\cos\phi > 1$, not a real number solution.

There are no integral multiples of N that equal M for $\Delta < 180°$.