

Solution to Problem 87

The problem can be rewritten as

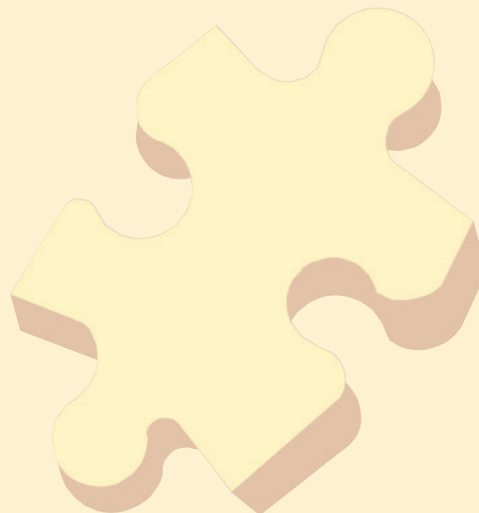
$$x = 1 + \frac{1}{x}$$

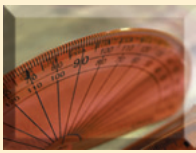
from which $x^2 - x - 1 = 0$

By the quadratic equation: $x = \frac{1 \pm \sqrt{(1 + 4)}}{2}$

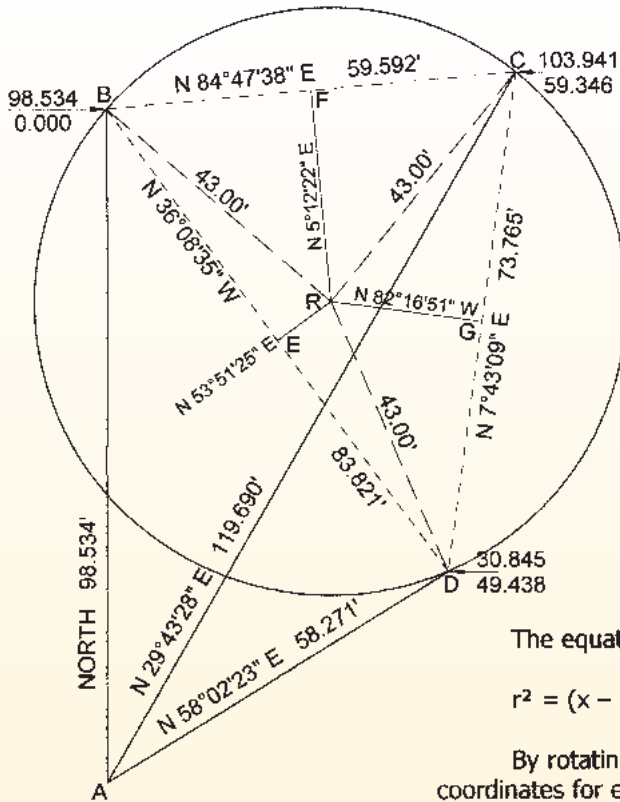
yielding $x = 1.618033989$ and $x = -0.618033989$

The first x is known as the "Golden Ratio" supposedly used in a lot of ancient Greek architecture.





Solution No. 1 to Problem 88



The equation of a circle whose center is at (N_0, E_0) is

$$r^2 = (x - E_0)^2 + (y - N_0)^2$$

By rotating the given data so that A-B is "North" and calculating coordinates for each reading with point A being $(0,0)$ we can write three equations for the circle:

- (1) $r^2 = x^2 + (y - 98.534)^2$
- (2) $r^2 = (x - 59.346)^2 + (y - 103.941)^2$
- (3) $r^2 = (x - 49.438)^2 + (y - 30.845)^2$

- (1) $r^2 = x^2 + y^2 - 197.068 y + 9708.949156$
- (2) $r^2 = x^2 - 118.692 x + 3521.947716 + y^2 - 207.882 y + 10,803.73148$
- (3) $r^2 = x^2 - 98.876 x + 2444.115844 + y^2 - 61.69 y + 951.414025$

Subtracting (1) from (2):

$$(4) \quad 118.692 x = -10.814 y + 4616.73004, \text{ \& } x = -0.091109763 y + 38.89672463$$

Subtracting (2) from (3)

$$(5) \quad 0 = 19.816 x + 146.192 y - 10,930.14933$$

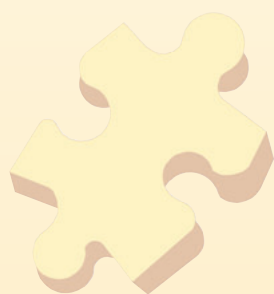
Substituting x from (4) into (5)

$$0 = 19.816(-0.091109763 y + 38.89672463) + 146.192 y - 10,930.14933$$

from which $y = 70.3623$, and substituted into (4) gives $x = 32.486$

substituting both into (1) yields $r = 43.000'$

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Solution No. 2 to Problem 88

The radius point of the circle will be at the intersection of the perpendicular bisectors of any two chords. The 3 points given will yield three chords: B-C, C-D, and B-D, allowing a check on your work.

Rotate the data so line A-B is "north" (not a necessity, but convenient), and calculate coordinates for each of the located points, i.e. B(98.534, 0.00), C(103.941, 59.346) and D(30.845, 49.438). Inversing,

$$B \text{ to } C = N 84^{\circ}47'38'' W \ 59.592'$$

$$C \text{ to } D = S 7^{\circ}43'09'' W \ 73.765'$$

$$D \text{ to } B = N 36^{\circ}08'35'' W \ 83.821'$$

The midpoints of the chords have coordinates E(64.689, 24.719), F(101.238, 29.673) and G(67.392, 54.392). You can calculate these by bearing and half the chord length or just average the northings and then the eastings.

A bearing-bearing intersection from any two midpoints at right angles to the chord will yield coordinates of the radius point. The possibilities are from E-F, F-G, and E-G. All three combinations give a coordinate of R(70.362, 32.486).

$$\text{Inversing from R to B, C, and D gives } r = 43.00'$$

SOLUTION NO. 3 TO PROBLEM NO. 88

Since the circle is the circumscribing circle of triangle B-C-D, its radius may be found by dividing any side of the triangle by twice the sine of the angle opposite:

$$r = \frac{B-C}{2 \sin(36^{\circ}08'35'' + 7^{\circ}43'09'')} = \frac{59.592}{1.385853169} = 43.00$$

$$r = \frac{C-D}{2 \sin(180^{\circ} - 84^{\circ}47'38'' - 36^{\circ}08'35'')} = \frac{73.765}{1.71546719} = 43.00$$

$$r = \frac{B-D}{2 \sin(84^{\circ}47'38'' - 7^{\circ}43'09'')} = \frac{83.821}{1.949325212} = 43.00$$

