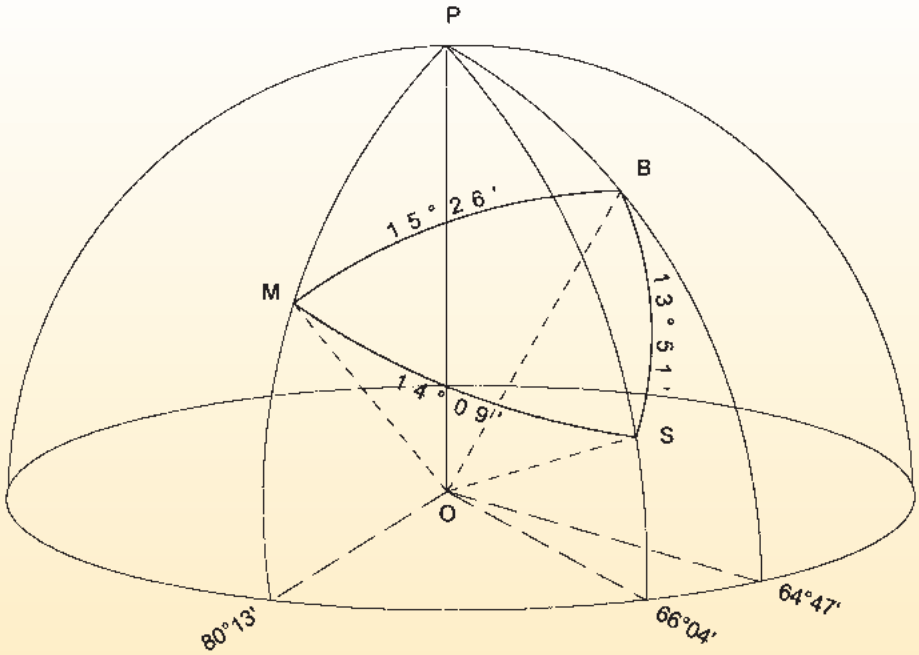
Solution to Problem 85



By differences of latitudes and longitudes, the sides of triangle M-B-S are shown in the diagram (sides are defined as the angle subtended at the center of the sphere).

Calculate the angles of triangle M-B-S by the law of cosines of spherical trigonometry:

$$\cos M = \frac{\cos 13°51' - \cos 15°26' \times \cos 14°09'}{\sin 15°26' \times \sin 14°09'}$$, so that M = 56°09'

$$\cos B = \frac{\cos 14°09' - \cos 15°26' \times \cos 13°51'}{\sin 15°26' \times \sin 13°51'}$$, so that B = 58°01'

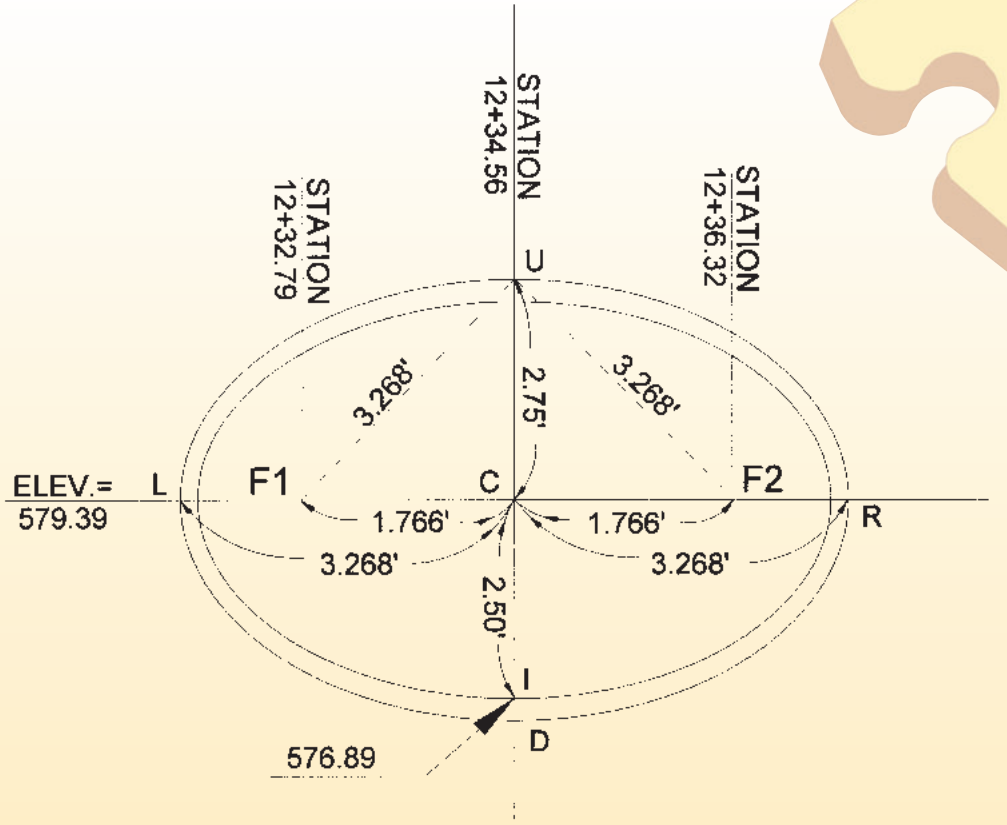
$$\cos S = \frac{\cos 15°26' - \cos 13°51' \times \cos 14°09'}{\sin 13°51' \times \sin 14°09'}$$, so that S = 67°25'

The three angles add up to 181°35' which is 1°35' of spherical excess, E.

The area of a spherical triangle is Er^2, where E is in radians, and the radius of the earth, r, is 3960 miles.

Area = $\pi r^2 E / 180 = \pi \times 3960^2 \times 1.583333 / 180 = 433{,}350$ square miles.

Solution to Problem 86



The horizontal component and major axis of the ellipse is determined by the angle of intersection and the outside diameter of the pipe. It is 66" / cos 32°42'15" = 78.434" or 6.536'.

The vertical component and minor axis of the ellipse is determined by the slope. There is no significant increase in the vertical direction, so the minor axis is 66" or 5.50'.

In the diagram, L-R is the major axis, U-D is the minor axis.

The center of the ellipse is 2.50' above the invert, or at elevation 579.39.

U-F1 = U-F2 = LR / 2 = 3.268'.

UC = CD = 2.75', therefore, by the Pythagorean Theorem: F1-C = C-F2 = 1.766' and the focal points are at station 12 + 32.79 and 12 + 36.32 and elevation 579.39.

To layout the ellipse with piano wire a length equal to F1-F2-U is required: 10.068'.