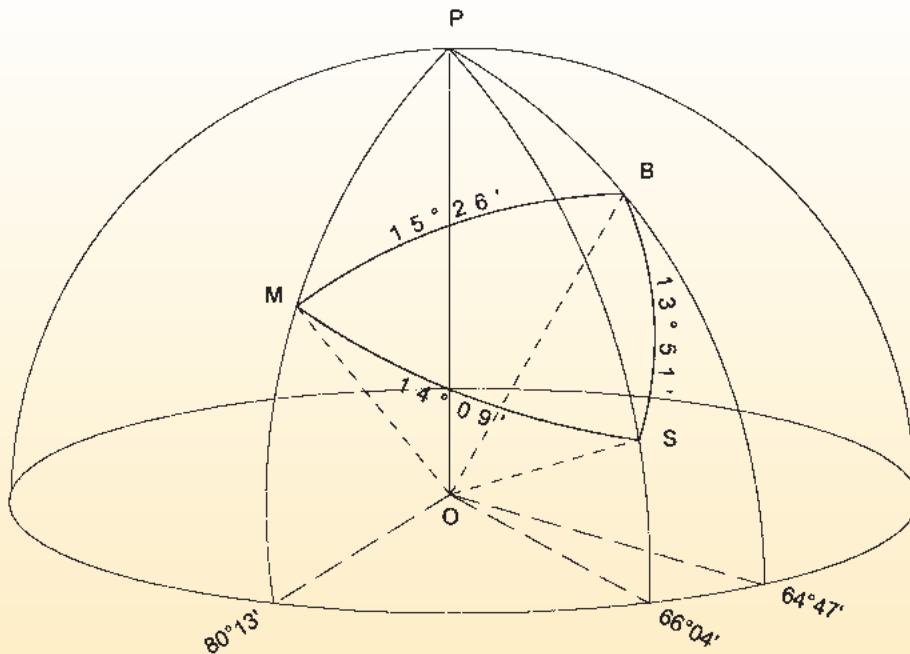




## Solution to Problem 85



By differences of latitudes and longitudes, the sides of triangle M-B-S are shown in the diagram (sides are defined as the angle subtended at the center of the sphere).

Calculate the angles of triangle M-B-S by the law of cosines of spherical trigonometry:

$$\cos M = \frac{\cos 13^{\circ}51' - \cos 15^{\circ}26' \times \cos 14^{\circ}09'}{\sin 15^{\circ}26' \times \sin 14^{\circ}09'}, \text{ so that } M = 56^{\circ}09'$$

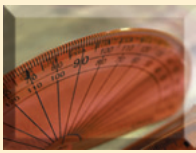
$$\cos B = \frac{\cos 14^{\circ}09' - \cos 15^{\circ}26' \times \cos 13^{\circ}51'}{\sin 15^{\circ}26' \times \sin 13^{\circ}51'}, \text{ so that } B = 58^{\circ}01'$$

$$\cos S = \frac{\cos 15^{\circ}26' - \cos 13^{\circ}51' \times \cos 14^{\circ}09'}{\sin 13^{\circ}51' \times \sin 14^{\circ}09'}, \text{ so that } S = 67^{\circ}25'$$

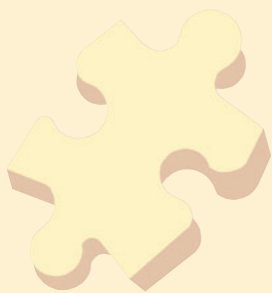
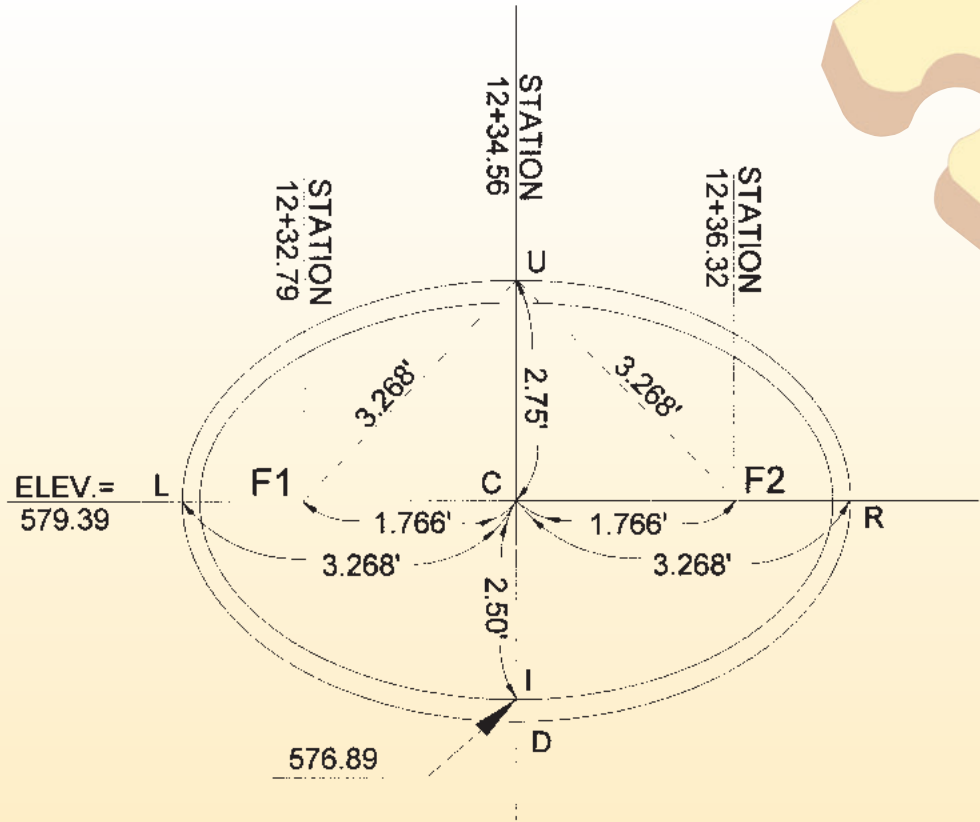
The three angles add up to  $181^{\circ}35'$  which is  $1^{\circ}35'$  of spherical excess, E.

The area of a spherical triangle is  $Er^2$ , where E is in radians, and the radius of the earth, r, is 3960 miles.

$$\text{Area} = \pi r^2 E / 180 = \pi \times 3960^2 \times 1.583333 / 180 = 433,350 \text{ square miles.}$$



# Solution to Problem 86



The horizontal component and major axis of the ellipse is determined by the angle of intersection and the outside diameter of the pipe. It is  $66'' / \cos 32^{\circ}42'15'' = 78.434''$  or  $6.536'$ .

The vertical component and minor axis of the ellipse is determined by the slope. There is no significant increase in the vertical direction, so the minor axis is  $66''$  or  $5.50'$ .

In the diagram, L-R is the major axis, U-D is the minor axis.

The center of the ellipse is  $2.50'$  above the invert, or at elevation  $579.39$ .

$U-F_1 = U-F_2 = LR / 2 = 3.268'$ .

$UC = CD = 2.75'$ , therefore, by the Pythagorean Theorem:  $F_1-C = C-F_2 = 1.766'$  and the focal points are at station  $12 + 32.79$  and  $12 + 36.32$  and elevation  $579.39$ .

To layout the ellipse with piano wire a length equal to  $F_1-F_2-U$  is required:  $10.068'$ .