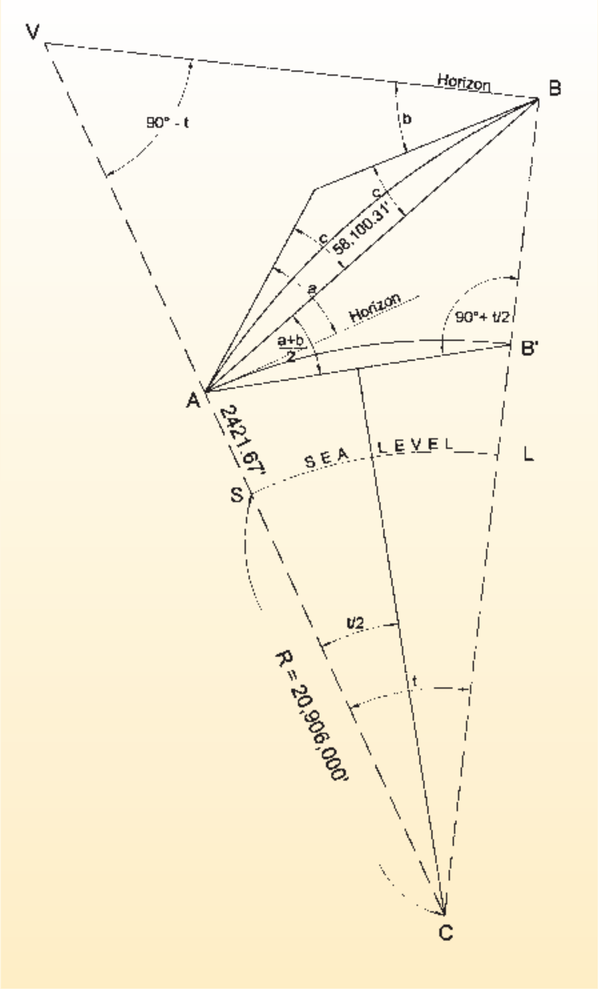
Solution to Problem 83



FROM THE DIAGRAM, $a = 2°27'01"$, $b = 2°29'22"$, c= REFRACTION CORRECTION, t= ANGLE SUBTENDED AT THE CENTER OF THE EARTH OF RADIUS 20,906,000'.

IN TRIANGLE A-V-B: $(b+c)+(90°-t)+[(90°-a)+c] = 180°$, $c = a/2 - b/2 + t/2$

ANGLE B-A-B' $= a - c + t/2 = a - a/2 + b/2 - t/2 + t/2 = (a + b)/2$

ANGLE A-B-B' + ANGLE B-A-B' $= 90° - t/2$

ANGLE A-B-B' $= 90° - t/2 - (a+b)/2$

AB' (THE CHORD) $= 2 R_{AB'} \sin t/2$, SO $\sin t/2 = A\text{-}B'/2R_{AB'}$, AND

AB' $= AB\,[\cos(a + b)/2]$ (APPROXIMATELY), AND

$$\sin t/2 = \frac{AB \cos[(a + b)/2]}{2R_{AB'}} = 0.001388109 \quad \text{AND } t/2 = 0°04'46.3"$$

BY THE LAW OF SINES:

$$\frac{AB'}{\sin [90° - t/2 - (a+b)/2]} = \frac{AB}{\sin (90° + t/2)}$$

$$\text{SO THAT CHORD AB'} = \frac{AB \sin [90° - t/2 - (a+b)/2]}{\sin (90° + t/2)} =$$

$$\frac{58100.31 \sin [90° - 0°04'46.3" - (2°27'01" + 2°29'22")/2]}{\sin (90° + 0°04'46.3")}$$

$$= \frac{58100.31 \times \sin 87°27'02.2"}{\sin 90°04'46.3"} = \frac{58100.31 \times 0.999010251}{0.999999037} = 58{,}042.861'$$

TO REDUCE CHORD AB' TO CHORD SL:

$$\frac{SL}{AB'} = \frac{2\,(R - \text{ELEV. OF A}) \sin t/2}{2 R \sin t/2} = \frac{R - 2421.67}{R}$$

SL = AB' – 2421.67 AB' / 20,906,000 = 58,042.861 – 6.723 = 58,036.138'

TO GET THE SEA LEVEL LENGTH OF ARC SL:

CHORD SL $= 2R \sin t/2 = 2R(t/2 - t^3/48 + t^5/3840\ldots)$ WHERE t IS IN RADIANS

ARC SL $= R\,t = 2\,R\,t/2$, ALSO WITH t IN RADIANS

ARC SL – CHORD SL $= 2\,R\,t/2 - 2\,R\,t/2 + 2\,R\,t^3/48 - 2\,R\,t^5/3840$

t IS SO SMALL THAT $t^5/3840$ CAN BE IGNORED,

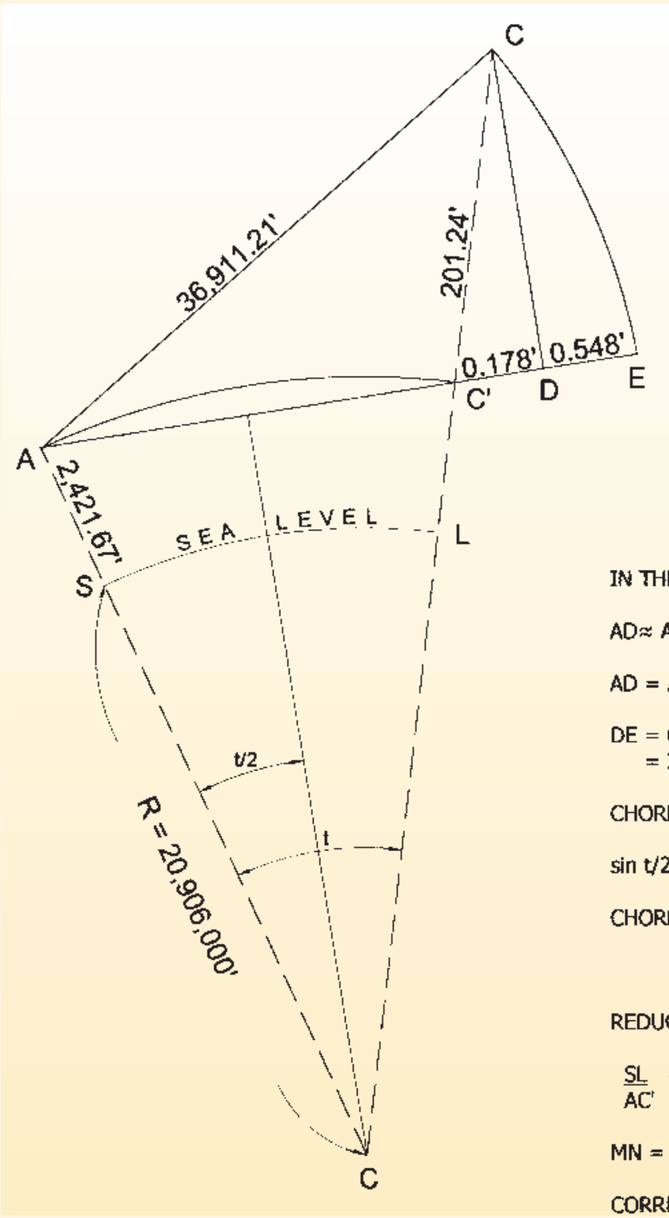
SO ARC SL – CHORD SL $= R\,t^3/24$, BUT t = ARC SL / R,

SO ARC SL – CHORD SL $= R\,SL^3 / 24\,R^3 = SL^3/24\,R^2$, THE DIFFERENCE BETWEEN THE ARC AND THE CHORD, TO BE ADDED TO THE CHORD:

$58{,}036.138^3/24 \times 20{,}906{,}000^2 = 0.019'$

AND ARC SL = 58,036.138' + 0.019' = 58,036.16'

Solution to Problem 84



IN THE DIAGRAM, A-S = 2,421.67', C-C' = 201.24', AC = AE

AD ≈ AC, CC' ≈ CD

AD = AE – DE = AC – DE

DE = $CD^2/2AC + CD^4/8AC^3$, OF WHICH THE SECOND TERM < 0.000005
= 201.24^2 / 2 x 36911.21 = 0.549'

CHORD AC' = 2(20,906,000 + 2421.67) sin t/2 = 41,816,843.34 sin t/2

sin t/2 = 36,911.21 / 41,816,843.34 = 0.000882688, AND t/2 =0°03'02"

CHORD AC' = AC – (C'D + DE)
= 36,911.21 – (201.24 x sin 0°03'02" + 201.24^2/2x36,911.21)
= 36,911.21 – 0.178 – 0.548 = 36,910.484

REDUCING CHORD AC' TO CHORD SL:

$$\frac{SL}{AC'} = \frac{2(R - \text{ELEVATION OF A})\sin t/2}{2\,R\,\text{SIN}\,t/2} = \frac{R - 2421.67}{R}$$

MN = AC' –36,910.484x2421.67/20,906,000 = 36,910.484 – 4.276 = 36,906.208

CORRECTION FROM CHORD TO ARC: $SL^3/24R^2$

$36,906.208^3$/ 24 x $20,906,000^2$ = 0.005'

SEA LEVEL DISTANCE = 36,906.208 +0.005 = 36,906.21'