



Solution to Problem 79

The solution requires finding “primitive Pythagorean triples,” with restrictions. Any primitive Pythagorean triple can be found using $p^2 - q^2$, $2p q$, $p^2 + q^2$ where $p > q$ and p and q are any two numbers.

The rules are, in addition to being integers:

- (a) $p^2 + q^2 \leq 100$, so $p \leq 9$, otherwise $q = 0$. ($p^2 + q^2$ must generate a number that is an integer on a 100 foot tape, hence less than or equal to 100)
- (b) $2pq \leq 100$ (the product of twice $p q$ must also generate an integral number on a 100 foot tape)
- (c) $2pq > 0$, so $p q > 0$ and $p = 2$, $q = 1$ is the lowest combination (which gives the well known $2^2 - 1^2 = 3$, $2 \times 2 \times 1 = 4$, $2^2 + 1^2 = 5$ triple)

also

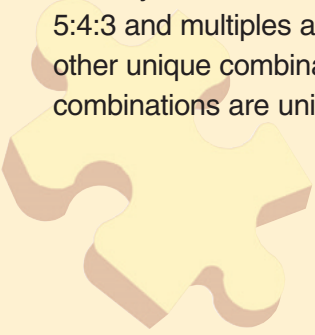
- (d) if p and q are relatively prime, only primitives result
- (e) if not both are odd, primitives result.

Let $q = 1$ while p changes from 2 to 9 [Rule (a)], then let $q = 2$ while p changes from 3 to 9, and so on until $q = 8$ and $p = 9$ [which is eliminated by Rule (a)]. There are $8+7+6+5+4+3+2+1$ combinations of p and q with these rules.

Numbers like 30:40:50 and 30:72:78 are multiples of 3:4:5 and 5:12:13, respectively, and are not unique. There are actually 104 combinations, but they include 3:4:5, 3:5:4, 4:3:5, 4:5:3, 5:3:4, 5:4:3 and multiples and rearrangements of the other unique combinations. The following 16 combinations are unique:



q	p	skipped by Rule	$p^2 - q^2$	$2 p q$	$p^2 + q^2$
1	2	3	4	5
1	3	(e)			
1	4	15	8	17
1	5	(e)			
1	6	35	12	57
1	7	(e)			
1	8	63	16	65
1	9	(e)			
2	3	5	12	13
2	4	(d)			
2	5	21	20	29
2	6	(d)			
2	7	45	28	53
2	8	(d)			
2	9	77	36	85
3	4	7	24	25
3	5	(e)			
3	6	(d)			
3	7	(e)			
3	8	55	48	73
3	9	(e,d)			
4	5	9	40	41
4	6	(d)			
4	7	33	56	65
4	8	(d)			
4	9	65	72	97
5	6	11	60	61
5	7	(d)			
5	8	39	80	89
5	9	(e)			
6	7	13	84	85
6	8	(d)			
6	9	(a,d)			
7	8	(a)			
7	9	(e)			
8	9	(a)			





Solution to Problem 80

B.S.	H.I.	F.S.	ELEV.	DESCR.
			345.675	BENCH MARK
4.835	<u>350.51</u>			
		<u>7.33</u>	343.18	0+00 TC
		<u>8.01</u>	342.50	0+00 FL
		<u>7.30</u>	343.21	0+00 C/L
		6.13	344.38	T.P.
<u>4.37</u>	348.75			
		4.27	<u>344.48</u>	1+00 TC
		4.90	<u>343.85</u>	1+00 FL
		4.21	<u>344.54</u>	1+00 C/L
		<u>5.96</u>	342.79	T.P.
3.59	346.38			
		<u>0.71</u>	345.67	1+50 TC
		1.38	<u>345.00</u>	1+50 FL
		0.68	345.70	1+50 C/L
		11.19	<u>335.19</u>	T.P.
2.14	<u>337.33</u>			
		8.49	<u>328.84</u>	B.M. CHK (328.823)

