Solution to Problem 75

You can find the lengths of the sides from Problem No. 54, or calculate them:
The area of the triangle = 1 acre = 43,560 sq. ft. = ½ ABxAC, but
AB = 0.75 AC, so (2)(43,560) = (0.75 AC)(AC), or 87120 = $0.75AC^2$
from which AC = 340.822' and AB = 255.617' and BC = 426.028'.

The length of the rope must be 213.014' + 340.822" – x (going counter-clockwise) and 213.014' + 255.617' + x (going clockwise), from which x = 42.602' and the length is 511.234'

Alternatively, twice the length must be 340.822'+426.028'+255.617' (the perimeter of the triangle) and the length is 511.234'.

The central angle of Area I is 180°–arctan 0.75=180°–36°52'12" or 143°07'48"

The central angle of Area III is 180° – arctan 1.33333... = 180° – 53°07'48" or 126°52'12"

The grazing area consists of four parts;

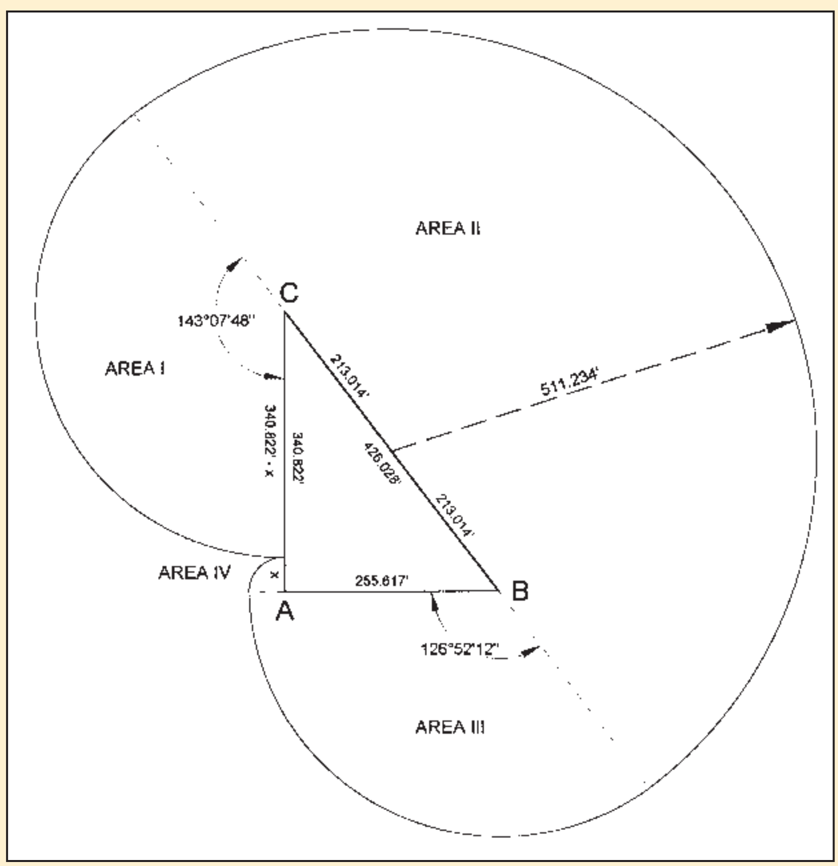
Area I = (143°07'48" / 360°) x π x 298.22^2 = 111,084.02 sq. ft.

Area II = ½ x π x 511.234^2 = 410,543.65 sq. ft.

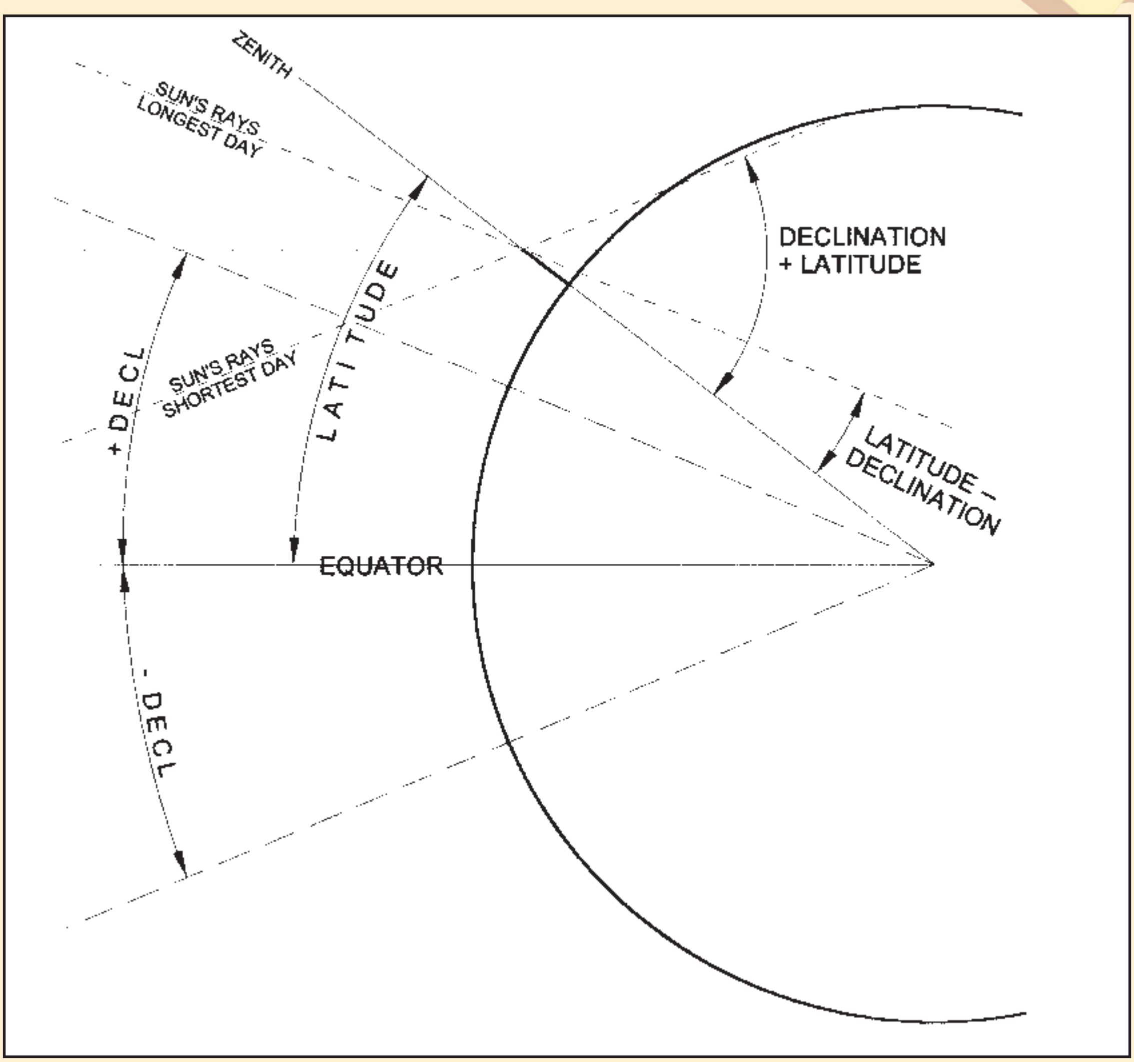
Area III = (126°52'12" / 360°) x π x 298.22^2 = 98,464.54 sq. ft.

Area IV = ¼ x π x 42.602^2 = 1,425.44 sq. ft.

for a total of 621,517.64 sq. ft. or 14.268 Acres.



Solution to Problem 76
Case I



(CASE I) WHERE THE LATTITUDE IS GREATER THAN THE DECLINATION AND THE SHADOWS ARE NORTH OF THE FLAGPOLE:

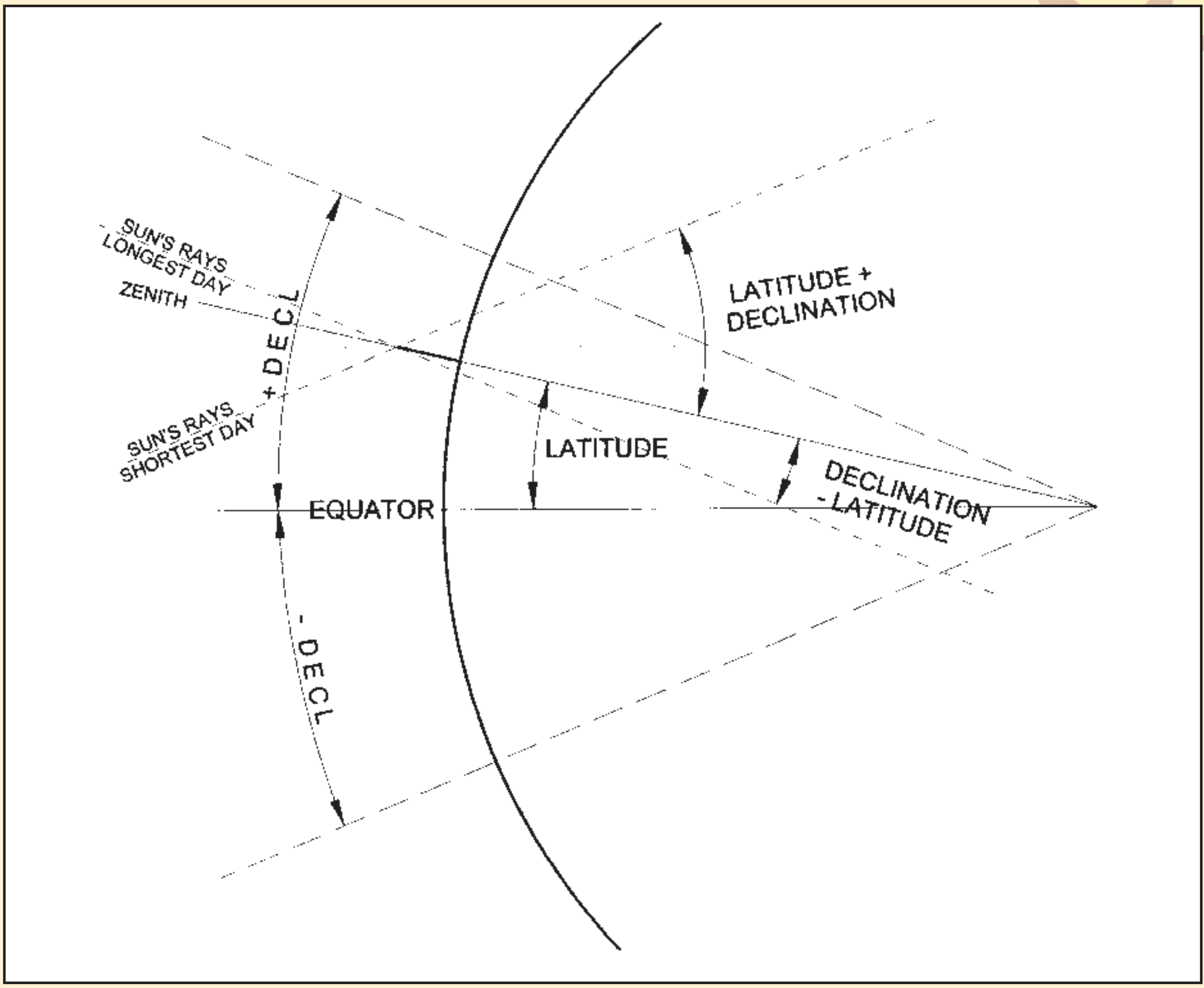
Let ϕ = latitude and δ = declination and H = height of flagpole.
From the drawing, $H \tan(\phi + \delta) = 8 H (\phi - \delta)$ or

$\tan(\phi + 23°28') = 8 \tan(\phi - 23°28')$. Equating by tangent of sums,

$$\frac{\tan\phi + \tan 23°28'}{1 - \tan\phi \tan 23°28'} = \frac{8(\tan\phi - \tan 23°28')}{(1 + \tan\phi \tan 23°28')}$$

Expanding and rearranging yields $9K\tan^2\phi - 7(1+K^2)\tan\phi + 9K = 0$,
where $K = \tan 23°28' = 0.434120782$, from which ϕ = 55°02'01" and 34°57'59"

Solution to Problem 76
Case 2



(CASE II) WHERE THE LATITUDE IS LESS THAN THE DECLINATION AND ONE SHADOW IS NORTH OF THE FLAGPOLE AND THE OTHER IS SOUTH:

$8 H \tan(\delta - \phi) = H \tan(\phi + \delta)$

$8 \tan(23°28' - \phi) = \tan(\phi + 23°28')$. Equating to tangents of sums,

$$\frac{8(\tan 23°28' - \tan \phi)}{1 + \tan 23°28' \tan \phi} = \frac{\tan \phi + \tan 23°28'}{1 - \tan \phi \tan 23°28'}$$

Expanding and rearranging yields

$7K \tan^2 \phi - 9(K^2 + 1) \tan \phi + 7K = 0$, where $K = \tan 23°28' = 0.434120782$, from which $\phi = 17°18'46"$ (The other solution, $72°41'13"$, gives a physically impossible answer as the altitude of the sun would be $-6°09'\pm$)