Solution to Problem 67

The most probable value is given by a least squares solution.
Let $L_1 = AB$, $L_2 = BC$, $L_3 = CD$ with residuals r_1, r_2 and r_3, respectively.

Set up condition equations:

$AB + r_1 = L_1$, so $r_1 = L_1 - AB = L_1 - 49.897$

$BC + r_2 = L_2$, so $r_2 = L_2 - BC = L_2 - 99.846$

$CD + r_3 = L_3$, so $r_3 = L_3 - CD = L_3 - 99.931$

$AC + r_1 + r_2 = L_1 + L_2$, so $r_1 + r_2 = L_1 + L_2 - AC = L_1 + L_2 - 149.797$

$BD + r_2 + r_3 = L_2 + L_3$, so $r_2 + r_3 = L_2 + L_3 - BD = L_2 + L_3 - 199.812$

To get a least squares solution the sum of the squares of the residuals must be minimized, to wit:

$\phi = r_1^2 + r_2^2 + r_3^2 + (r_1 + r_2)^2 + (r_2 + r_3)^2 = (L_1 - 49.897)^2 + (L_2 - 99.846)^2 + (L_3 - 99.931)^2 + (L_1 + L_2 - 149.797)^2 + (L_2 + L_3 - 199.812)^2$ must be minimized.

The partial derivative evaluated with respect to each estimate and equated to zero will minimize ϕ:

$$\frac{\partial \phi}{\partial L_1} = 2(L_1 - 49.897) + 2(L_1 + L_2 - 149.797) = 0$$

$$\frac{\partial \phi}{\partial L_2} = 2(L_2 - 99.846) + 2(L_1 + L_2 - 149.797) = 0$$

$$\frac{\partial \phi}{\partial L_3} = 2(L_3 - 99.931) + 2(L_2 + L_3 - 199.812) = 0$$

$L_1 - 49.897 + L_1 + L_2 - 149.797 = 0$, or $2 L_1 + L_2 = 199.694$

$L_2 - 99.846 + L_1 + L_2 - 149.797 + L_2 + L_3 - 199.812 = 0$, or $L_1 + 3 L_2 + L_3 = 499.455$

$L_3 - 99.931 + L_2 + L_3 - 199.812 = 0$, or $L_2 + 2 L_3 = 299.743$

$2 L_1 + 6 L_2 + 2 L_3 - 2 L_1 - L_2 = 699.216 = 5 L_2 + 2 L_3$

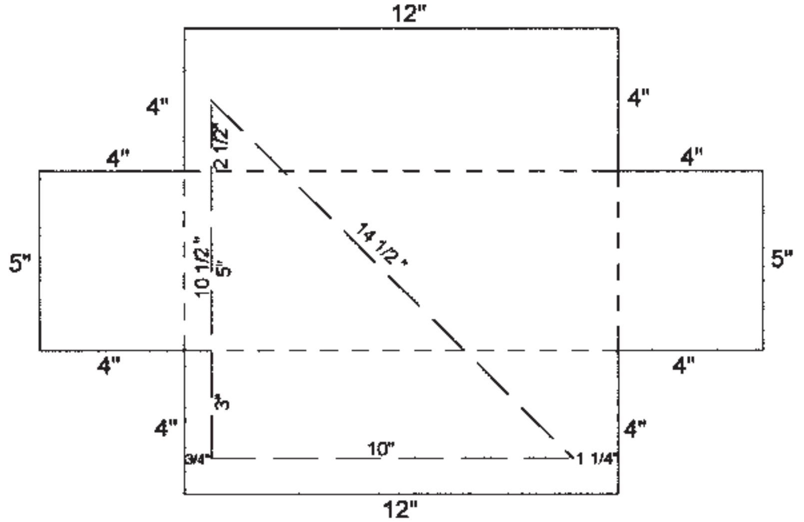
but $5 L_2 + 10 L_3 = 1498.715$, so $8 L_3 = 799.499$ and $L_3 = 99.9374$

From which $L_1 = 49.9129$, $L_2 = 99.8683$ and A-D = 249.7185

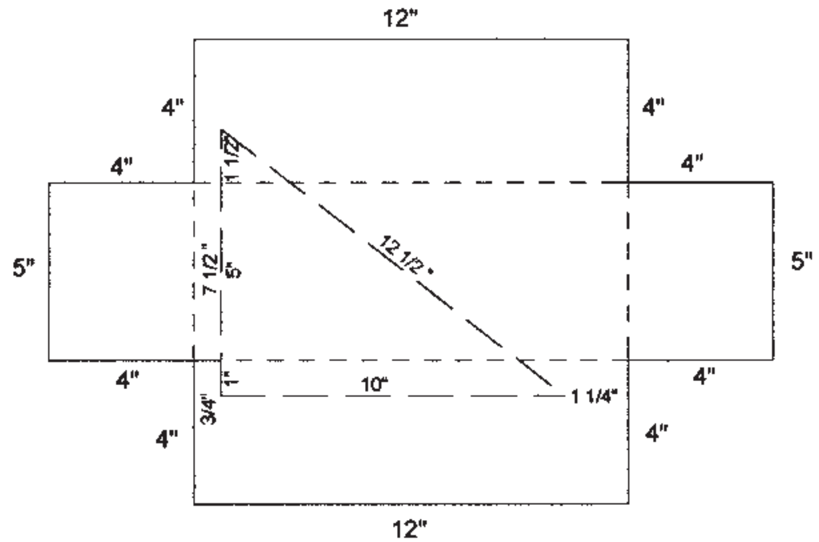
Solution to Problem 68

"OPEN" THE BOX SO IT LAYS FLAT. THERE ARE TWO POSSIBILITIES.

DOWN THE WALLS AND ACROSS THE FLOOR:



UP THE WALLS AND ACROSS THE CEILING:



THE CEILING ROUTE IS SHORTER