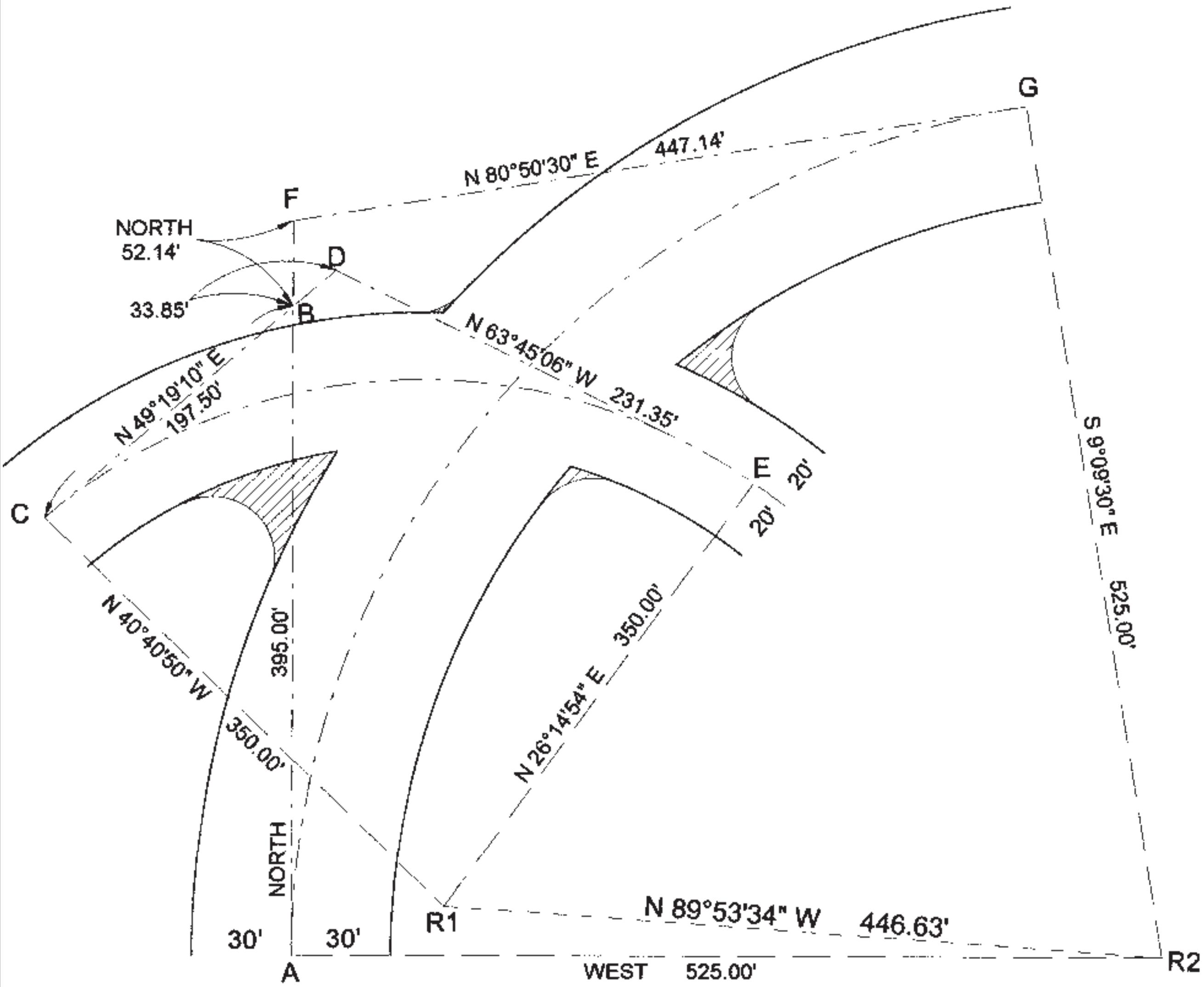
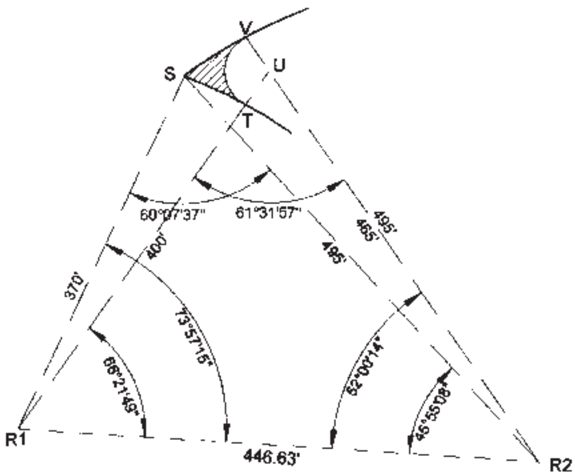
Solution to Problem 63



Begin by traversing from one radius point to the other using the given data and calculated semi-tangents. The inverse distance between the radius points will be used in several calculations. (I assumed "north" for one semi-tangent of the 525' radius curve. Use whatever you like. Bearings are not used in the solution, only distances.)

Solution to Problem 63 *continued*



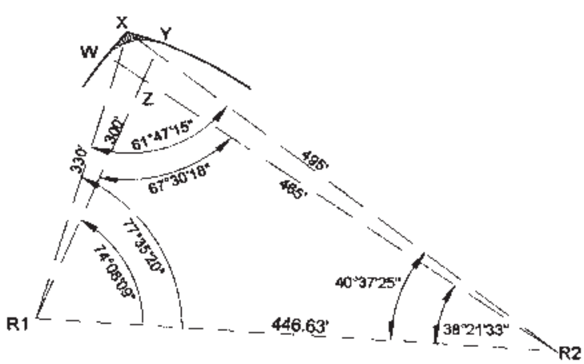
Easterly corner:

By law of cosines, solve triangle R1-R2-S (with sides of 446.63',370' and 495') for the three angles as shown.

Then solve triangle R1-R2-U (with sides of 446.63', 400' and 465') for the three angles as shown.

Differences of angles gives central angles for sectors R1-S-T (7°35'26") and R2-S-V (6°05'06").

The area of triangle R1-S-R2 (79,407.538 sq. ft.), plus the area of sector R2-S-V (13,011.230 sq. ft.),minus the area of sector R1-S-T (9,068.269 sq. ft.),minus the area of sector T-U-V (930.442 sq. ft.), minus the area of triangle R1-U-R2 (81,832.384 sq. ft.) is the area of the cut-off (587.67 sq. ft., the shaded portion in the drawing).



Southeasterly corner (this is a tricky one!):

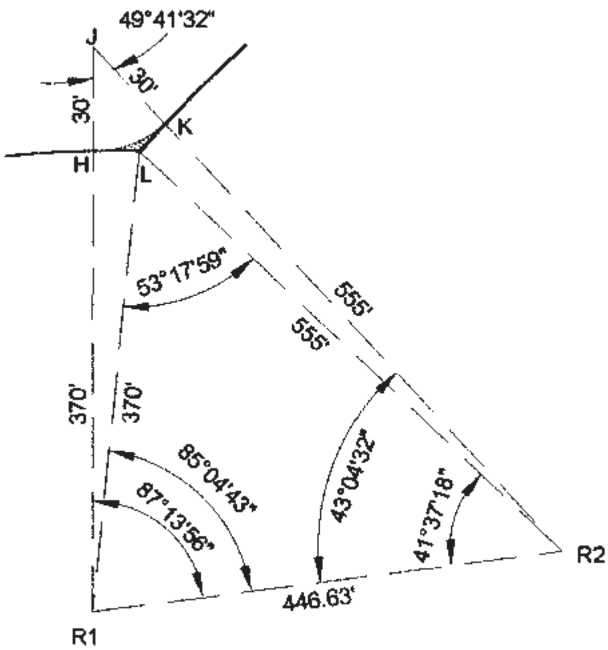
By law of cosines, solve triangle R1-X-R2 (with sides 330', 495' and 446.63') for the three angles as shown.

Then solve triangle R1-Z-R2 (with sides 300', 465' and 446.63') for the three angles as shown.

Again differences of angles give central angles for sectors R1-X-Y (3°27'12") and R2-W-X (2°15'52").

The area of sector R1-X-Y (3,281.812 sq. ft.), plus the area of triangle R1-Z-R2 (64,442.879 sq. ft.),plus the area of sector R2-W-X (4,841.940 sq. ft.), minus the area of triangle R1-X-R2 (71,971.837 sq. ft.), minus the area of sector W-Z-Y (530.183 sq. ft.) is the area of the cut-off (64.61 sq. ft., the shaded portion in the drawing).

Solution to Problem 63 *continued*



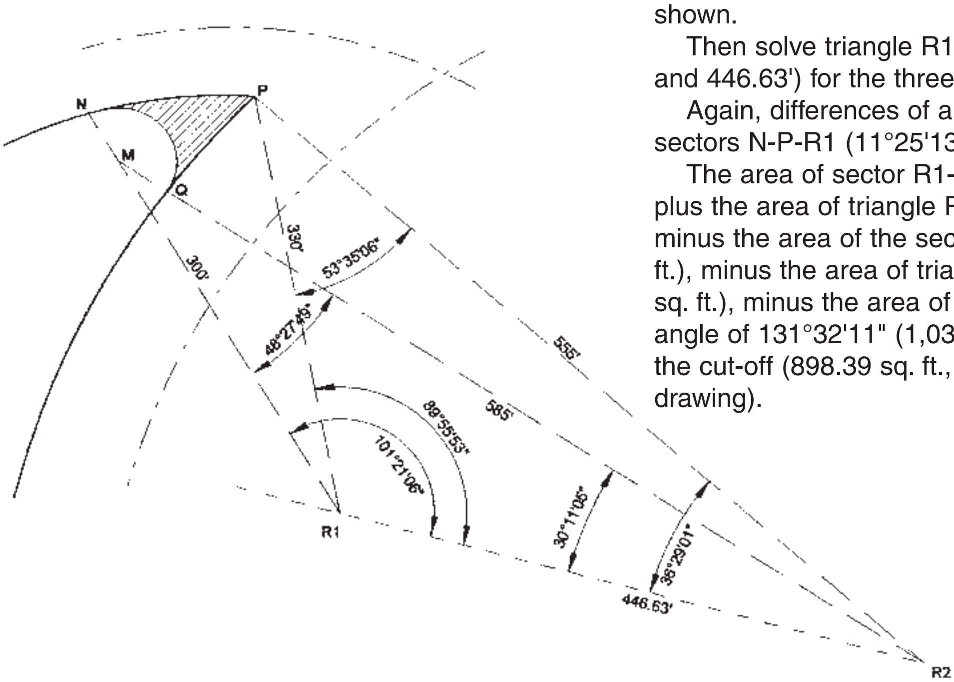
Northerly corner:

By law of cosines, solve triangle R1-R2-J (with sides of 446.63', 585' and 400') for the three angles as shown.

Then solve triangle R1-R2-L (with sides of 446.63', 555' and 370') for the three angles as shown.

Differences of angles gives central angles for sectors R1-H-L (2°09'13") and R2-L-K(1°27'14").

The area of triangle R1-J-R2 (89,222.799 sq. ft.), minus the area of sector R1-H-L (2,572.872 sq. ft.), minus the area of sector J-H-K (390.282 sq. ft.), minus the area of sector R2-L-K (3,908.090 sq. ft.), minus the area of triangle R1-L-R2 (82,321.942 sq. ft.) is the area of the cut-off (28.61 sq. ft., the shaded portion in the drawing).



Southwesterly corner:

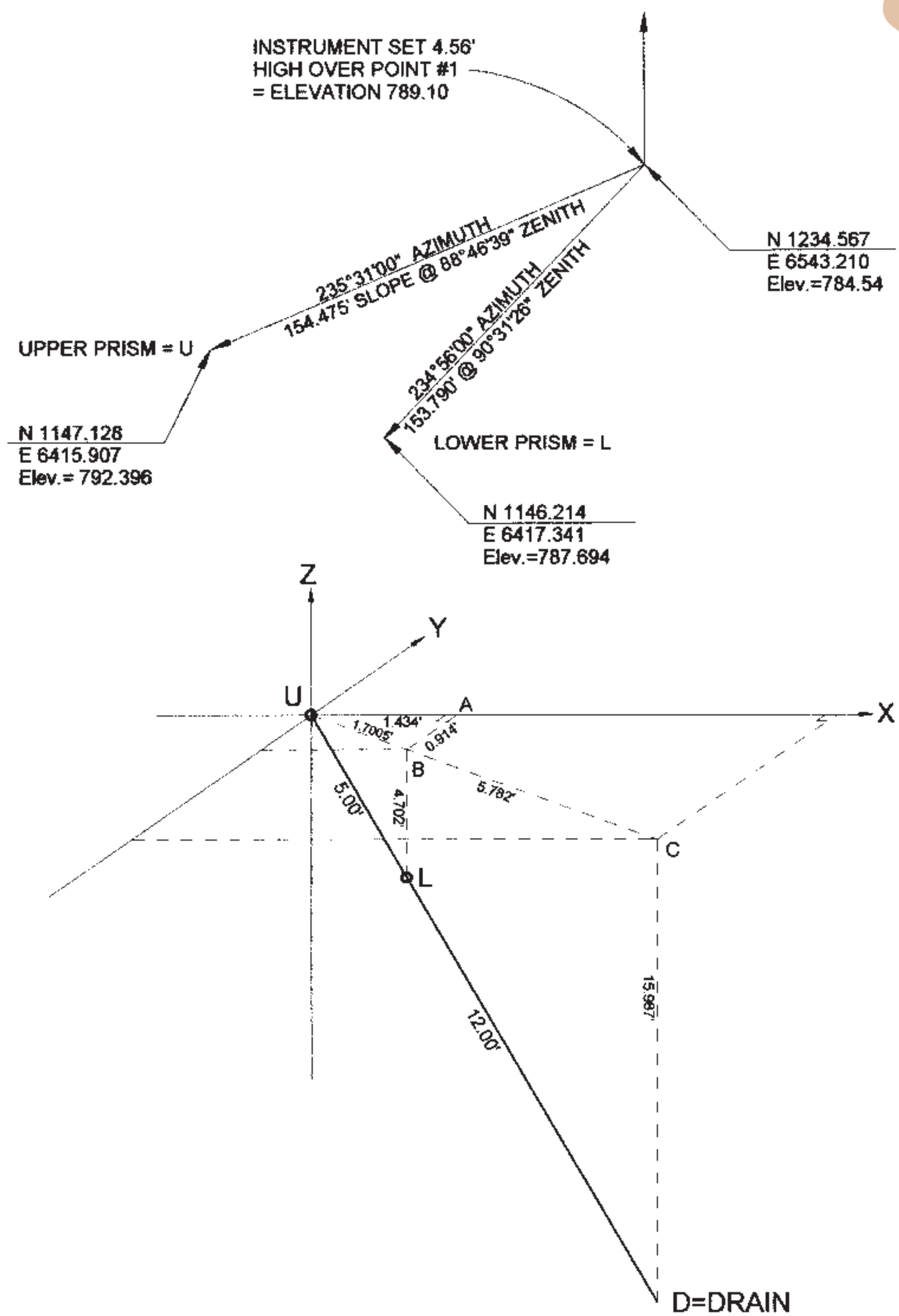
By law of cosines, solve triangle R1-M-R2 (with sides 300', 585' and 446.63') for the three angles as shown.

Then solve triangle R1-P-R2 (with sides 330', 555' and 446.63') for the three angles as shown.

Again, differences of angles give central angles for sectors N-P-R1 (11°25'13") and Q-P-R2 (6°17'56").

The area of sector R1-N-P (10,853.053 sq. ft.), plus the area of triangle R1-P-R2 (73,693.897 sq. ft.), minus the area of the sector R2-Q-P (16,931.572 sq. ft.), minus the area of triangle R1-M-R2 (65,683.907 sq. ft.), minus the area of sector M-N-Q with a central angle of 131°32'11" (1,033.084 sq. ft.) is the area of the cut-off (898.39 sq. ft., the shaded portion in the drawing).

Solution to Problem 64



Solution to Problem 64 *continued*

Calculate the northing, easting and elevation for each prism, as shown.

Construct a three dimensional coordinate system through the upper prism so the x-axis points East and the y-axis points North. The z-axis represents elevations.

$UB^2 = UA^2 + AB^2$; $UB^2 = 1.434^2 + 0.914^2$, UB = 1.7005'

By proportion: $\frac{UB}{5} = \frac{UC}{17}$, UC = 5.782' ; $\frac{CD}{17} = \frac{BL}{5}$, CD = 15.987'

Angle A-B-U = arc tan 1.434 / 0.914 = 57°29'15" so that the drain is S 57°29'15" E 5.782' from U with an elevation 15.987' lower or North 1144.02, East 6420.78, elevation 776.41'.

Alternatively, using *direction cosines*:

Let U be (x_1, y_1, z_1), L be (x_2, y_2, z_2) and the drain be (x_3, y_3, z_3)

If α, β, and γ represent the angles of a line from the x-, y-, and z-axes, then

$$\cos\alpha = \frac{x_2 - x_1}{d} \;;\; \cos\beta = \frac{y_2 - y_1}{d} \;;\; \cos\gamma = \frac{z_2 - z_1}{d},$$ where d is the line length.

$\cos\alpha = 1.434/5 = 0.2868$, $\cos\beta = -0.914/5 = -0.1828$, $\cos\gamma = -4.702/5 = -0.9404$

since line U-L and U-D have the same direction cosines,

$$\cos\alpha = \frac{x_3 - x_1}{17} \;;\; \cos\beta = \frac{y_3 - y_1}{17} \;;\; \cos\gamma = \frac{z_3 - z_1}{17}$$

$(17)\cos\alpha + x_1 = x_3$; $(17)\cos\beta + y_1 = y_3$ and $(17)\cos\gamma + z_1 = z_3$

so, $x_3 = 6420.783$, $y_3 = 1144.020$ and $z_3 = 776.409$

making the drain North 1144.02, East 6420.78 at elevation 776.41'

