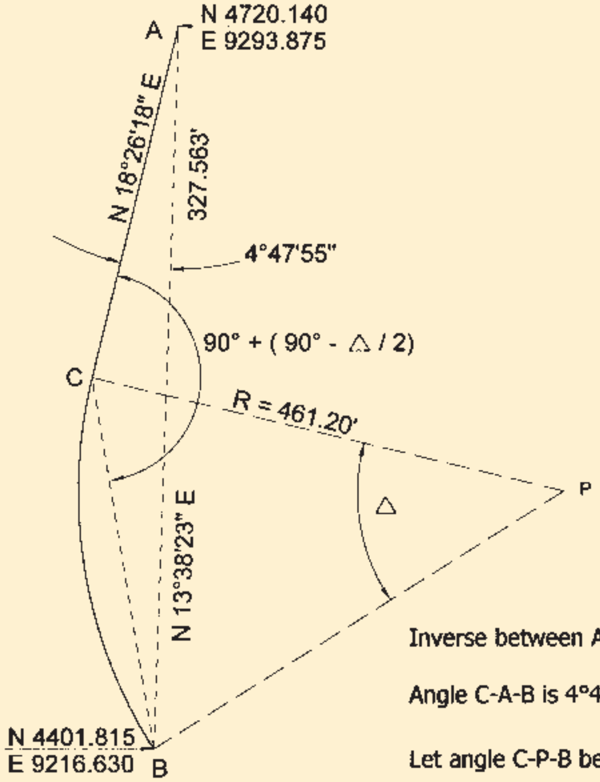
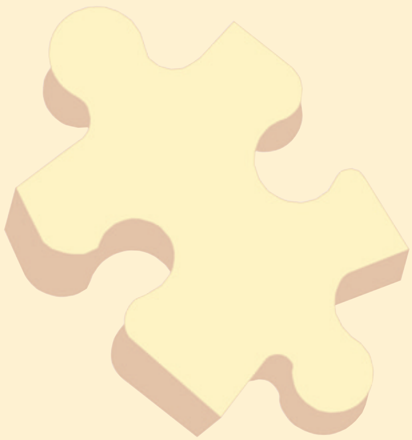
Solution to Problem 61





Inverse between A and B to get N 13°38'23" E 327.563'.

Angle C-A-B is 4°47'55", by difference of bearings.

Let angle C-P-B be Δ.

Angle A-C-B is then 90° + (90° - Δ / 2).

B-C is the chord subtended by Δ and is equal to 2 R sin (Δ/2), where R = 461.20'

By the Law of Sines:

$$\frac{[2 R \sin (\Delta/2)]}{\sin 4°47'55"} = \frac{327.563}{\sin (180° - \Delta/2)}$$

$922.40 \times \sin^2 \Delta/2 = 27.40185288$

$\sin^2 \Delta/2 = 0.029707126$

$\sin \Delta/2 = 0.172357552$

From which $\Delta = 19°50'59"$

The chord B-C = 158.983' and angle A-C-B = (90° + 90° - 9°55'30") = 170°04'30"

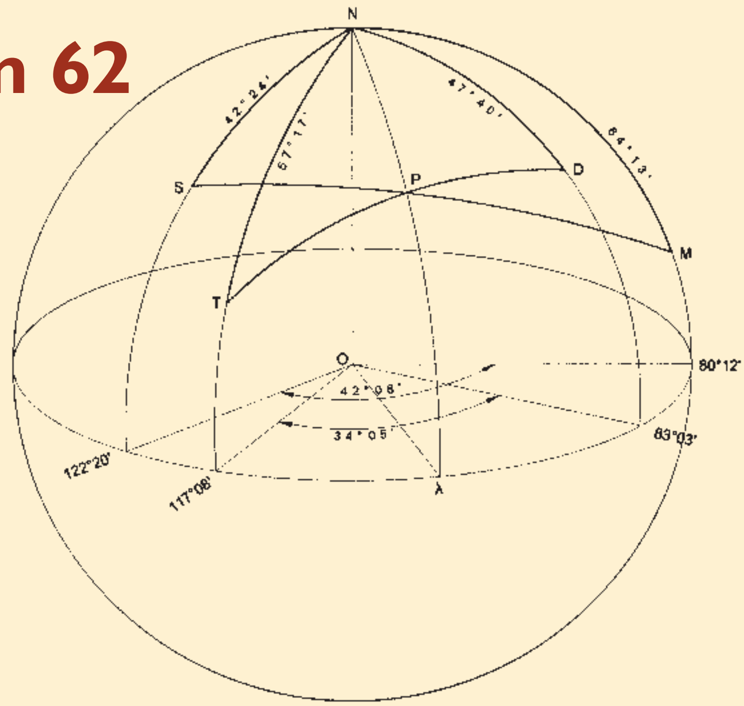
Angle C-B-A is 180° – 4°47'55" – 170°04'30" = 5°07'35"

Again, by Law of Sines;

$$\frac{A\text{-}C}{\sin 5°07'35"} = \frac{158.983}{\sin 4°47'55"}$$

from which A-C = 169.814'

Solution to Problem 62



From the Law of Cosines of spherical trigonometry:

In triangle SMN: $\cos SM = \cos SN \cos MN + \sin SN \sin MN \cos SNM$

$= \cos 42°24' \cos 64°13' + \sin 42°24' \sin 64°13' \cos 42°08'$

from which $SM = 39°30'48''$

$$\cos S = \frac{\cos MN - \cos SN \cos SM}{\sin SN \sin SM} = \frac{\cos 64°13' - \cos 42°24' \cos 39°30'48''}{\sin 42°24' \sin 39°30'48''}$$

from which $S = 108°18'10''$

$$\cos M = \frac{\cos SN - \cos SM \cos MN}{\sin SM \sin MN} = \frac{\cos 42°24' - \cos 39°30'48'' \cos 64°13'}{\sin 39°30'48'' \sin 64°13'}$$

from which $M = 45°18'50''$

In triangle TDN: $\cos TD = \cos TN \cos DN + \sin TN \sin DN \cos N$

$= \cos 57°17' \cos 47°40' + \sin 57°17' \sin 47°40' \cos 34°05'$

from which $TD = 28°27'54''$

$$\cos T = \frac{\cos DN - \cos TN \cos TD}{\sin TD \sin TN} = \frac{\cos 47°40' - \cos 57°17' \cos 28°27'54''}{\sin 28°27'54'' \sin 57°17'}$$

from which $T = 60°21'48''$

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$$\cos D = \frac{\cos TN - \cos TD \cos DN}{\sin TD \sin DN} = \frac{\cos 57°17' - \cos 28°27'54'' \cos 47°40'}{\sin 28°27'54'' \sin 47°40'}$$

from which D = 98°24'44"

Rotate the sphere so 0° longitude passes through Miami.

From the Law of Sines of spherical trigonometry:

$$\frac{\sin TP}{\sin (36°56' - \lambda)} = \frac{\sin (90° - \phi)}{\sin 60°21'48''} \quad \text{and} \quad \frac{\sin MP}{\sin \lambda} = \frac{\sin (90° - \phi)}{\sin 45°18'50''}$$

$$\sin TP \sin 60°21'48'' = \sin (36°56' - \lambda) \cos \phi \qquad 1(a)$$

$$\text{and} \quad \sin MP \sin 45°18'50'' = \sin \lambda \cos \qquad 1(b)$$

From the Law of Cosines:

$$\sin TP \cos 60°21'48'' = \cos(90° - \phi) \sin 57°17' - [\sin(90° - \phi) \cos 57°17' \cos(36°56' - \lambda)]$$

$$\sin TP \cos 60°21'48'' = \sin\phi \sin 57°17' - \cos\phi \cos 57°17' \cos(36°56' - \lambda) \qquad 2(a)$$

$$\text{and} \quad \sin MP \cos 45°18'50'' = \sin\phi \sin 64°13' - \cos \phi \cos 64°13' \cos \qquad 2(b)$$

Dividing 1(a) by 2(a) yields

$$\tan \phi = \frac{\sin(36°56' - \lambda) + \tan 60°21'48'' \cos 57°17' \cos (36°56' - \lambda)}{\sin 57°17' \tan 60°21'48''} \qquad 3(a)$$

Dividing 1(b) by 2(b) yields

$$\tan \phi = \frac{\sin \lambda + \tan 45°18'50'' \cos 64°13' \cos \lambda}{\sin 64°13' \tan 45°18'50''} \qquad 3(b)$$

Equating 3(a) and 3(b) and expanding the values in parentheses yields $\tan \lambda = 0.348575785$ for a $\lambda = 19°13'02''$, but that is from the 0° longitude through Miami. Adding the rotation of 80°12', $\lambda = 99°25'02''$

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Inserting the value for λ in equation 3(b)

$$\tan\phi = \frac{\sin 19°13'02 + \tan 45°18'50'' \cos 64°13' \cos 19°13'02''}{\sin 64°13' \tan 45°18'50''}$$

$\tan\phi = 0.81770126$, and $\phi = 39°16'22''$

(Inserting the value in equation 3(a) yields $\phi = 39°16'23''$. The precise value is 39°16'22.66984" and the precise longitude is 99°25'02.51623")