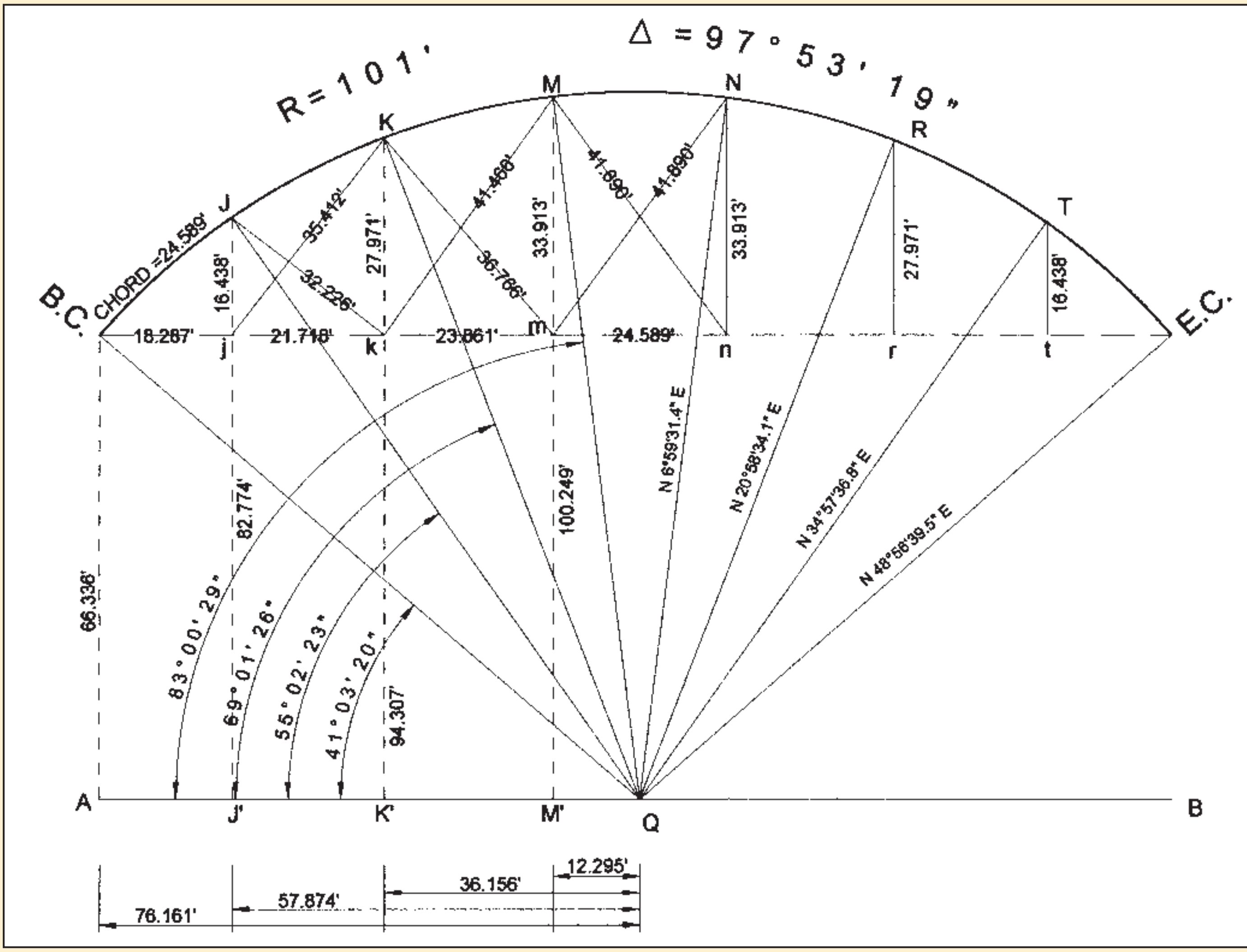
Solution to Problem 59



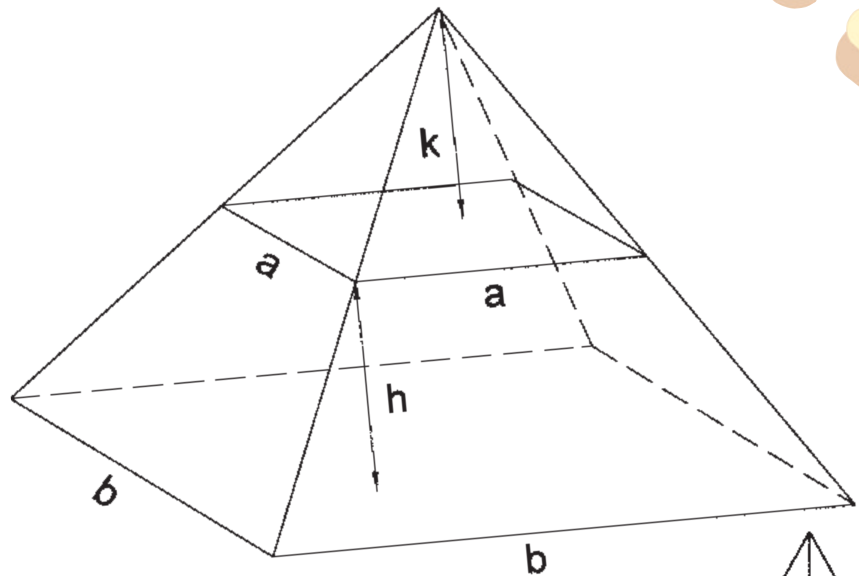
DRAW LINE AB PARALLEL WITH THE LONG CHORD (B.C TO E.C).
CONSTRUCT POINTS J, K, M, N, R, AND T, EACH AT 1/7 DELTA (13°59'02.7")
CONSTRUCT PERPENDICULARS FROM B, J, K AND M TO AB.

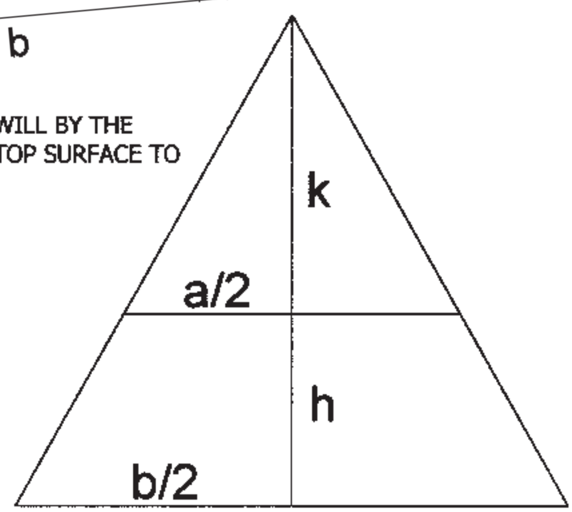
ANGLE E.C.-B.C.-Q = ANGLE A-Q-B.C. = 1/2(180°-97°53'19") = 41°03'20.5"
CALCULATE B.C.-A, J-J', K-K' AND M-M' USING THE SINE FUNCTION AND RADIUS
CALCULATE A-Q, J'-Q, K'-Q AND M'-Q USING THE COSINE FUNCTION AND RADIUS

J-j, K-k, M-m, B.C.-j, j-k and k-m ARE DERIVED BY SUBTRACTION
m-n IS TWICE M'-Q DUE TO SYMMETRY.

B.C.-J = 24.589 IS THE CHORD FOR 1/7 OF THE CENTRAL ANGLE.
CALCULATING DIAGONALS J-k, j-K, K-m, k-M AND M-n=m-N GIVES
ALL NECESSARY INFORMATION TO CONSTRUCT AND CHECK THE
CURVE IN THE FIELD FROM LINE B.C.-E.C.

Solution to Problem 60





EXTEND THE SIDES TO INTERSECT AT A POINT (THEY WILL BY THE DEFINITION OF A PYRAMID). LET THE DISTANCE FROM THE TOP SURFACE TO THE POINT BE "K"

$$\frac{k}{a/2} = \frac{k+h}{b/2}, \quad kb = ak + ah, \quad k(b-a) = ah, \quad k = \frac{ah}{b-a}$$

$$V = 1/3\, b^2(k+h) - 1/3 a^2 k$$

$$3V = b^2k + b^2h - a^2k = b^2 \frac{ah}{(b-a)} + b^2h - a^2 \frac{ah}{(b-a)}$$

$$= \frac{ab^2h}{(b-a)} + \frac{b^3h - ab^2h}{(b-a)} - \frac{a^3h}{(b-a)} = \frac{b^3h - a^3h}{b-a}$$

$$= \frac{h(b-a)(a^2 + ab + b^2)}{(b-a)}, \text{ and } V = 1/3\, h\,(a^2 + ab + b^2)$$

For our example, $a = 300$, $b = 480$, $h = 70$:

$V = (1/3)(70)(300^2 + 300 \times 480 + 480^2)$

$= (1/3)(70)(90{,}000 + 144{,}000 + 230{,}400) = 10{,}836{,}000$ cubic feet

$= 401{,}333$ cubic yards

IF THE SURFACES ARE IRREGULAR BUT PARALLEL THE FORMULA BECOMES

$V = 1/3\, h[(B_1 + (B_1B_2)^{1/2} + B_2]$, where B_1 is one base area and B_2 the other.

