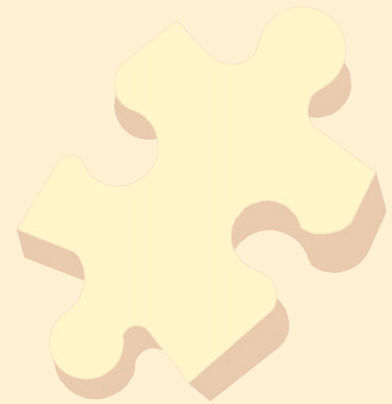
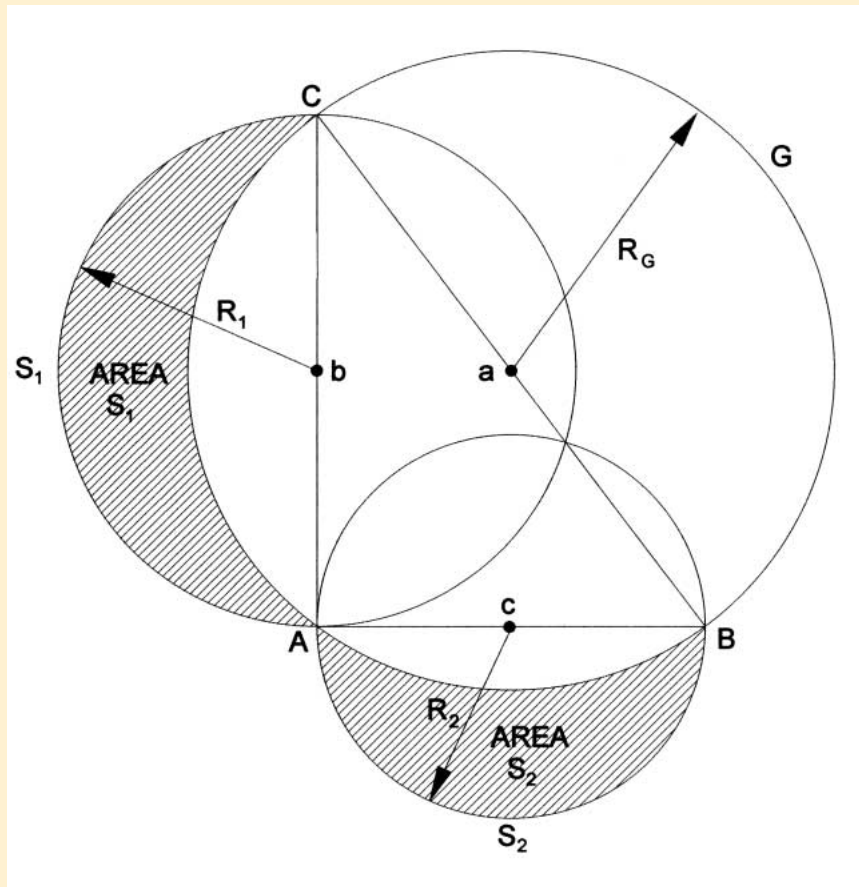




Solution to Problem 53



Area S_1 + area S_2 = area of semicircle $AcBS_2$ + area semicircle $AbCS_1$
+ area triangle ABC – area semicircle $aBAC$

$$= \frac{\pi R_2^2}{2} + \frac{\pi R_1^2}{2} + \frac{AC \times AB}{2} - \frac{\pi R_G^2}{2}$$

$$= \frac{\pi}{2} [(AB/2)^2 + (AC/2)^2 - (BC/2)^2] + \frac{AC \times AB}{2}$$

$$= \frac{\pi}{8} [(AB)^2 + (AC)^2 - (BC)^2] + \frac{AC \times AB}{2}$$

but $(AB)^2 + (AC)^2 = (BC)^2$, so $[(AB)^2 + (AC)^2 - (BC)^2] = 0$

and Area S_1 + area S_2 = $\frac{AC \times AB}{2}$, the area of the triangle, which is one acre.





Solution to Problem 54

The area of triangle ABC = 1 acre = 43,560 sq. ft = $\frac{1}{2} AB \times AC$,
 but $AB = 0.75 AC$, so $(2)(43560) = (0.75 AC)(AC)$ or $87120 = 0.75AC^2$

from which $AC = 340.822'$, $AB = 255.617'$ and $BC = 426.028'$ (By proportions)

$ab = \frac{1}{2} AB = 127.808'$, $Ab = bC = \frac{1}{2} AC = 170.411'$, and $Ca = aB = Aa = 213.014'$

$$\cos \text{angle } AaC = \frac{(aA)^2 + (aC)^2 - (AC)^2}{2(aA)(aC)} = \frac{213.014^2 + 213.014^2 - 340.822^2}{(2)(213.014)^2}$$

$$= -0.279996996 \text{ and angle } AaC = 106^\circ 15' 36''$$

Area S_1 = Area semicircle AS_1C + area triangle ACa – area sector AaC

$$\text{Area } S_1 = \frac{1}{2} R_1^2 + 127.808 \times 170.411 - (106^\circ 15' 36'' / 360^\circ)(213.014^2) \pi$$

$$= 45,615.925 + 21,780.000 - 42,075.939 = 25,319.986 \text{ sq. ft.}$$

Area S_2 = Area semicircle AS_2B + area triangle AaB – area sector AaB

$$\text{Area } S_2 = \frac{1}{2} R_2^2 + 127.808 \times 170.411 - (73^\circ 44' 24'' / 360^\circ)(213.014^2) \pi$$

$$= 25,658.958 + 21,780.000 - 29,198.944 = 18,240.014 \text{ sq. ft.}$$

Area S_1 + area S_2 = 25,319.986 sq. ft. + 18,240.014 sq. ft. = 43,560.000 sq. ft.

Area G = $\pi R_G^2 + 43,560 - \pi R_1^2 - \pi R_2^2 + \text{segment } Ab'Q + \text{segment } Ac'Q$

$$= \pi(R_G^2 - R_1^2 - R_2^2) + 43,560 + \text{segment } Ab'Q + \text{segment } Ac'Q$$

$$= 43,560 + (73^\circ 44' 24'' / 360^\circ) \pi (170.411^2) - 136.329 \times 102.247$$

$$+ (106^\circ 15' 36'' / 360^\circ) \pi (127.808^2) - 76.685 \times 102.247$$

$$= 55,614.66 \text{ sq. ft.}$$

