Solution to Problem 43

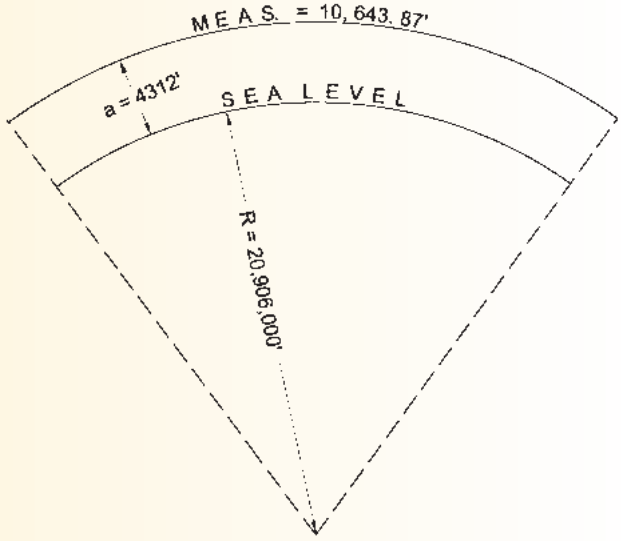
SOLUTION TO PROBLEM NO. 43 (TEXT)

Let the sea level distance be equal to the measured distance, m, less some correction, c.

Let R equal the radius of the earth at sea level and let a equal the average height.

$$\text{By similar sectors:} \quad \frac{m-c}{R} = \frac{m}{R+a}$$

$$\frac{m}{R} - \frac{c}{R} = \frac{m}{R+a} = m\left(\frac{1}{R} - \frac{a}{R^2} + \frac{a^2}{R^3} - \ldots\right)$$



$$c = \frac{ma}{R} - \frac{ma^2}{R^2} + \frac{ma^3}{R^3} - \ldots$$

$c = 2.19537' - 0.00045' + 0.0000001' \ldots = 2.195'$

sea level = $10,643.87' - 2.195' = 10.641.675'$

OR,

$$\frac{m}{R+a} = \frac{s}{R}, \text{ so } s = m\left(\frac{R}{R+a}\right) = 10,643.87' \frac{20,906,000}{(20,906,000 + 4312)} = 10,641.675'$$

Elevation differences become significant when $c = 0.005'$ or $100\left(\frac{R}{R+a}\right) = 0.99995$

$$c = 0.005' = \frac{100\,a}{20,906,000}, \text{ that is when } a = 1045'$$

OR,

$$\frac{R}{R+a} = 0.99995, \quad R - 0.99995\,R = 0.99995\,a, \quad a = 1045'$$

Note: R = 20,906,000' per *Surveying* by Bouchard & Moffit, Ninth edition, 1992, page 375.

Solution to Problem 44

SOLUTION TO PROBLEM NO. 44 (TEXT)

Let the highway elevation = 100.00′. The inner rail elevation is therefore 103.00′.
The outer rail elevation is 103.00′ plus the superelevation:

e (feet) = 0.000685 (80^2)(1.2) / 12 = 0.44′ , so the outer rail elevation is 103.44′.

The grade across the tracks is 0.44′/ 5′ = 0.088 , or 8.8%. In Figure 1, a change from 0% grade to +8.8% grade is

$$r = \frac{8.8 - 0}{0.682} = 12.9, \text{ but a comfortable } r = \frac{15{,}000}{70^2} = 3.06.$$

The transition must be made as shown in Figure 2.
The rate for L_1 cannot exceed $r = -3.06$ and L_2 cannot exceed $r = +3.06$

$$r_1 = -3.06 = \frac{g_1 - 0}{L_1} \text{ and } r_2 = 3.06 = \frac{8.8 - g_1}{L_2}$$

so $g_1 = -3.06\, L_1$ and $L_2 = L_1 + 2.8758$ (substituting g_1 in first equation into second)

also, $100.00' + g_1 (L_1/2 + L_2/2) + 8.8 (L_2/2) = 103.00'$

substituting $-3.06\, L_1$ for g_1 and (L_1 + 2.8758) for L_2 then expanding and rearranging yields

$L_1 = 1.7762$, $g_1 = -5.435\%$, and $L_2 = 4.6520$

Likewise, r_3 cannot exceed -3.06 and r_4 cannot exceed $+3.06$

$$r_3 = -3.06 = \frac{g_2 - 8.8}{L_3}, \text{ and } r_4 = 3.06 = \frac{0 - g_2}{L_4}$$

so $g_2 = -3.06\, L_4$ and $L_3 = L_4 + 2.8758$

and, $103.44' + 8.8 (L_3/2) + g_2(L_3/2 + L_4/2) = 100.00'$

again, substituting $-3.06\, L_4$ for g_2 and (L_4 + 2.8758) for L_3, expanding and rearranging,

$L_4 = 2.2933$, $L_3 = 5.1691$ and $g_2 = -7.0175\%$

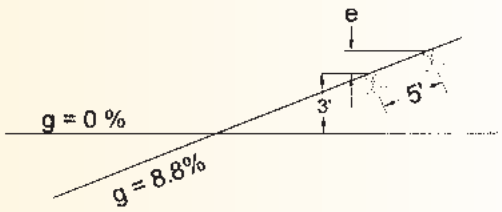


FIGURE 1

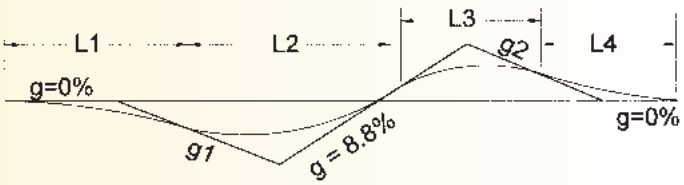


FIGURE 2