



Solution to Problem 43

SOLUTION TO PROBLEM NO. 43 (TEXT)

Let the sea level distance be equal to the measured distance, m , less some correction, c .

Let R equal the radius of the earth at sea level and let a equal the average height.

By similar sectors:

$$\frac{m - c}{R} = \frac{m}{R + a}$$

$$\frac{m - c}{R} = \frac{m}{R + a} \Rightarrow m - c = m \left(\frac{R}{R + a} \right)$$

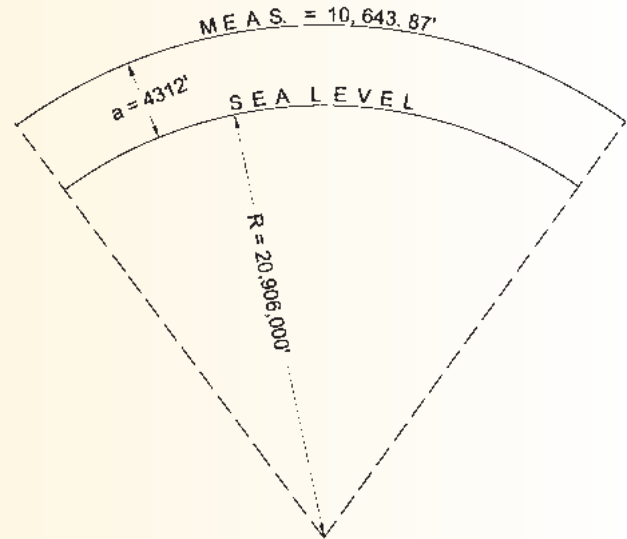
$$c = \frac{ma}{R} - \frac{ma^2}{R^2} + \frac{ma^3}{R^3} - \dots$$

$$c = 2.19537' - 0.00045' + 0.0000001' \dots = 2.195'$$

$$\text{sea level} = 10,643.87' - 2.195' = 10,641.675'$$

OR,

$$\frac{m}{R + a} = \frac{s}{R}, \text{ so } s = m \left(\frac{R}{R + a} \right) = 10,643.87' \frac{20,906,000}{(20,906,000 + 4312)} = 10,641.675'$$



Elevation differences become significant when $c = 0.005'$ or $100 \left(\frac{R}{R + a} \right) = 0.99995$

$$c = 0.005' = \frac{100 a}{20,906,000}, \text{ that is when } a = 1045'$$

OR,

$$\frac{R}{R + a} = 0.99995, R - 0.99995 R = 0.99995 a, a = 1045'$$

Note: $R = 20,906,000'$ per *Surveying* by Bouchard & Moffit, Ninth edition, 1992, page 375.





Solution to Problem 44

SOLUTION TO PROBLEM NO. 44 (TEXT)

Let the highway elevation = 100.00'. The inner rail elevation is therefore 103.00'. The outer rail elevation is 103.00' plus the superelevation:

$$e \text{ (feet)} = 0.000685 (80^2)(1.2) / 12 = 0.44', \text{ so the outer rail elevation is } 103.44'.$$

The grade across the tracks is $0.44' / 5' = 0.088$, or 8.8%. In Figure 1, a change from 0% grade to +8.8% grade is

$$r = \frac{8.8 - 0}{0.682} = 12.9, \text{ but a comfortable } r = \frac{15,000}{70^2} = 3.06.$$

The transition must be made as shown in Figure 2.

The rate for L_1 cannot exceed $r = -3.06$ and L_2 cannot exceed $r = +3.06$

$$r_1 = -3.06 = \frac{g_1 - 0}{L_1} \text{ and } r_2 = 3.06 = \frac{8.8 - g_1}{L_2}$$

so $g_1 = -3.06 L_1$ and $L_2 = L_1 + 2.8758$ (substituting g_1 in first equation into second)

$$\text{also, } 100.00' + g_1 (L_1 / 2 + L_2 / 2) + 8.8 (L_2 / 2) = 103.00'$$

substituting $-3.06 L_1$ for g_1 and $(L_1 + 2.8758)$ for L_2 then expanding and rearranging yields

$$L_1 = 1.7762, \quad g_1 = -5.435\%, \text{ and } L_2 = 4.6520$$

Likewise, r_3 cannot exceed -3.06 and r_4 cannot exceed $+3.06$

$$r_3 = -3.06 = \frac{g_2 - 8.8}{L_3}, \text{ and } r_4 = 3.06 = \frac{0 - g_2}{L_4}$$

so $g_2 = -3.06 L_4$ and $L_3 = L_4 + 2.8758$

$$\text{and, } 103.44' + 8.8 (L_3 / 2) + g_2 (L_3 / 2 + L_4 / 2) = 100.00'$$

again, substituting $-3.06 L_4$ for g_2 and $(L_4 + 2.8758)$ for L_3 , expanding and rearranging,

$$L_4 = 2.2933, \quad L_3 = 5.1691 \text{ and } g_2 = -7.0175\%$$

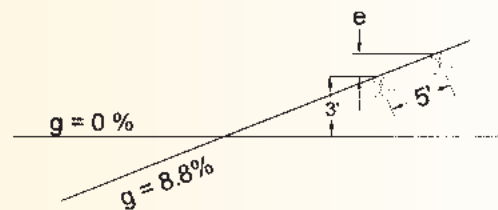


FIGURE 1

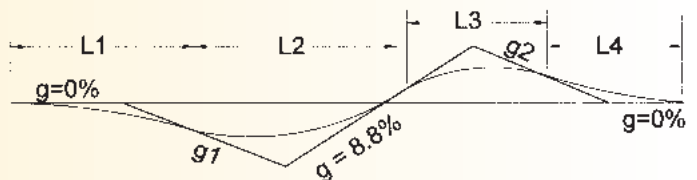


FIGURE 2