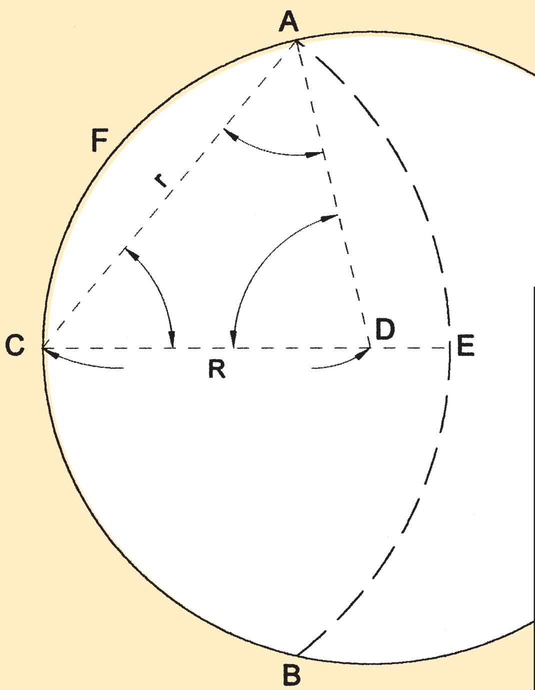
# Solution to Problem 32




To simplify the problem, let $R = AD = CD = 1$. Let angle ADC be $\phi$.

Angle ACD = angle CAD = $\frac{1}{2}(\pi - \phi)$, *with all angles expressed in radians.*

The area of sector A-C-E equals $\frac{1}{2} r^2 (\pi/2 - \phi/2)$.

The area of segment A-F-C equals $\frac{1}{2} R^2(\phi - \sin\phi)$, with $R = 1$.

The area of the sector plus the area of the segment must equal $\pi/4$.

so, $$\frac{r^2}{2}\,\frac{(\pi-\phi)}{2} + \frac{1}{2}(\phi - \sin\phi) = \frac{\pi}{4},$$

or, $\pi r^2 - \phi r^2 + 2\phi - 2\sin\phi - \pi = 0$

since $2R\sin(\phi/2) = r$, with $R = 1$, $r^2 = 4\sin^2(\phi/2) = 2(1 - \cos\phi)$

substituting for $r^2$,

$$\pi(2 - 2\cos\phi) - \phi(2 - 2\cos\phi) + 2\phi - 2\sin\phi - \pi = 0$$

which reduces to $\left(\phi - \pi - \tan\phi - \frac{\pi}{2\cos\phi}\right) = 0$, after collecting terms and

dividing by $2\cos\phi$.

Knowing that the value of $\phi$ has to be greater than zero and less than $\pi/2$ (that would make ADB a straight line), plotting a few values such as $\phi = 0$, 0.5, 1, and 1.5 shows the value to be near 1.25 ( There is an asymptotic line at $\phi = 1.5708\pm$ where $\phi = 90°$). Further refinement yields $\phi = 1.235896924$ and

$$r = 1.158728414\,R$$

( $\phi = 1.2359$ is within 1 second of arc for $\phi$ and within 0.0000025R for r)