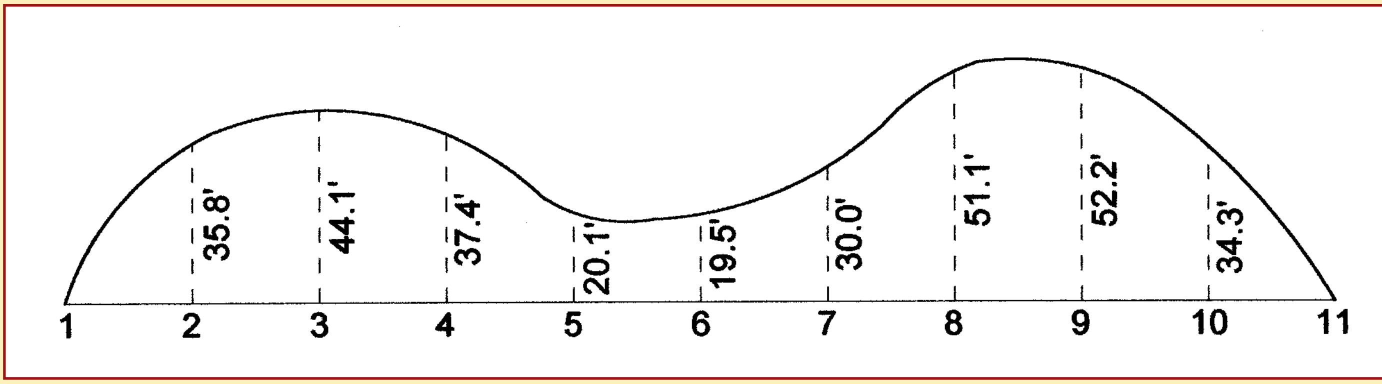
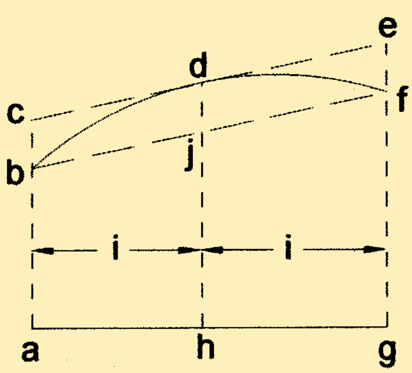
Solution to Problem 29





If we let arc bdf be part of a parabola, the area of segment bdfj can be shown to be 2/3 the area of parallelogram bcdefj. The area of any portion defined by three offsets is

$$\frac{ab + fg}{2} 2i + \frac{2}{3} 2i\,(dj)$$

$$\text{but } dj = dh - jh = dh - \frac{ab + fg}{2}$$

Applying this to 1-2-3, 3-4-5, 5-6-7, 7-8-9, and 9-10-11 in combination,

Any area, A = i / 3 (first offset + 4 times second offset + third offset)

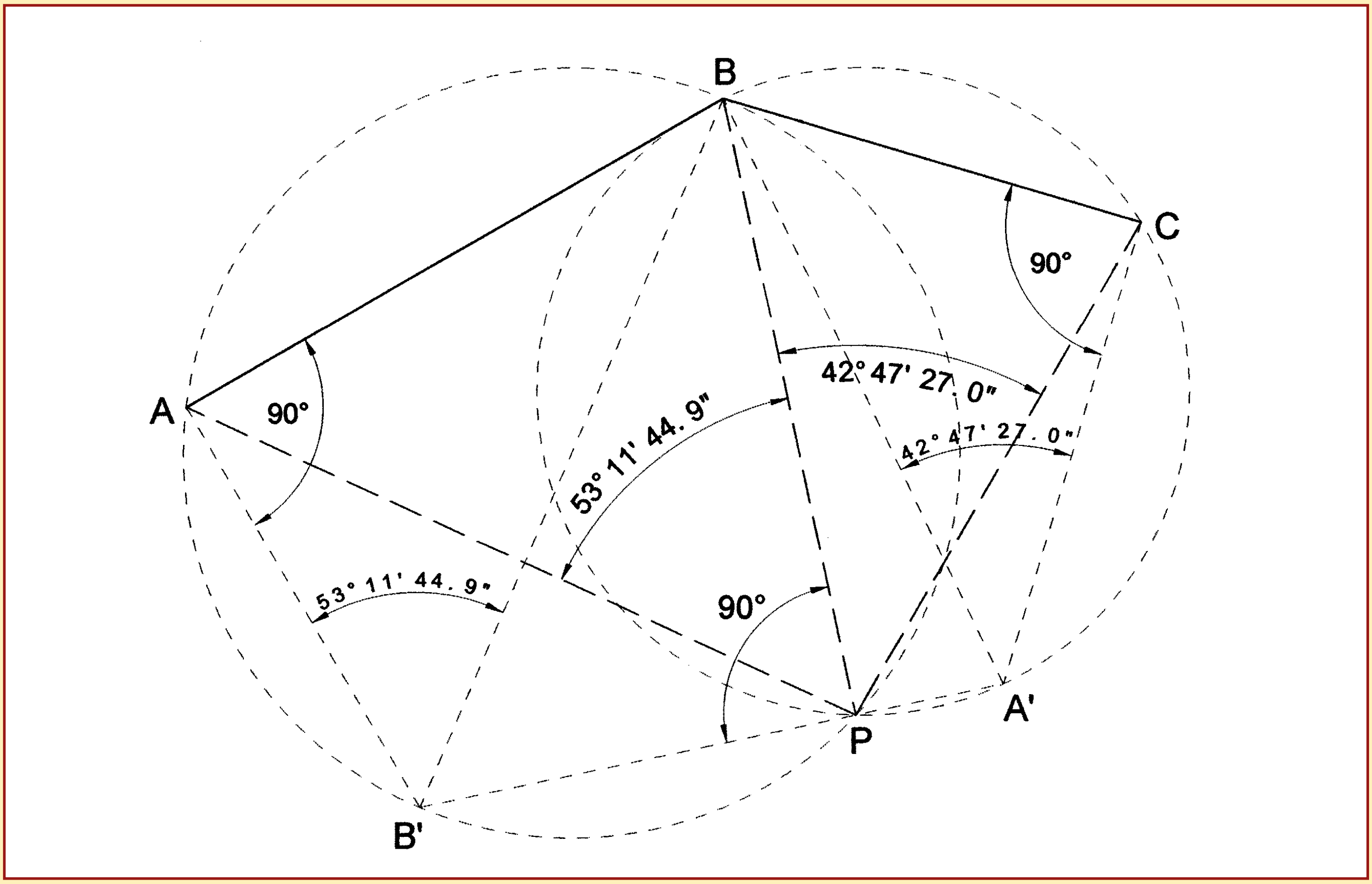
Summing all of the combinations,

the Area = i /3 (the first offset + 4 times the even numbered offsets + two times the odd numbered offsets + the last offset)

In this case,

$$A = \frac{28}{3} [\, 0 + (4)(178.1) + (2)\,(146.4) + 0 \,] = 9382 \text{ sq. ft.}$$

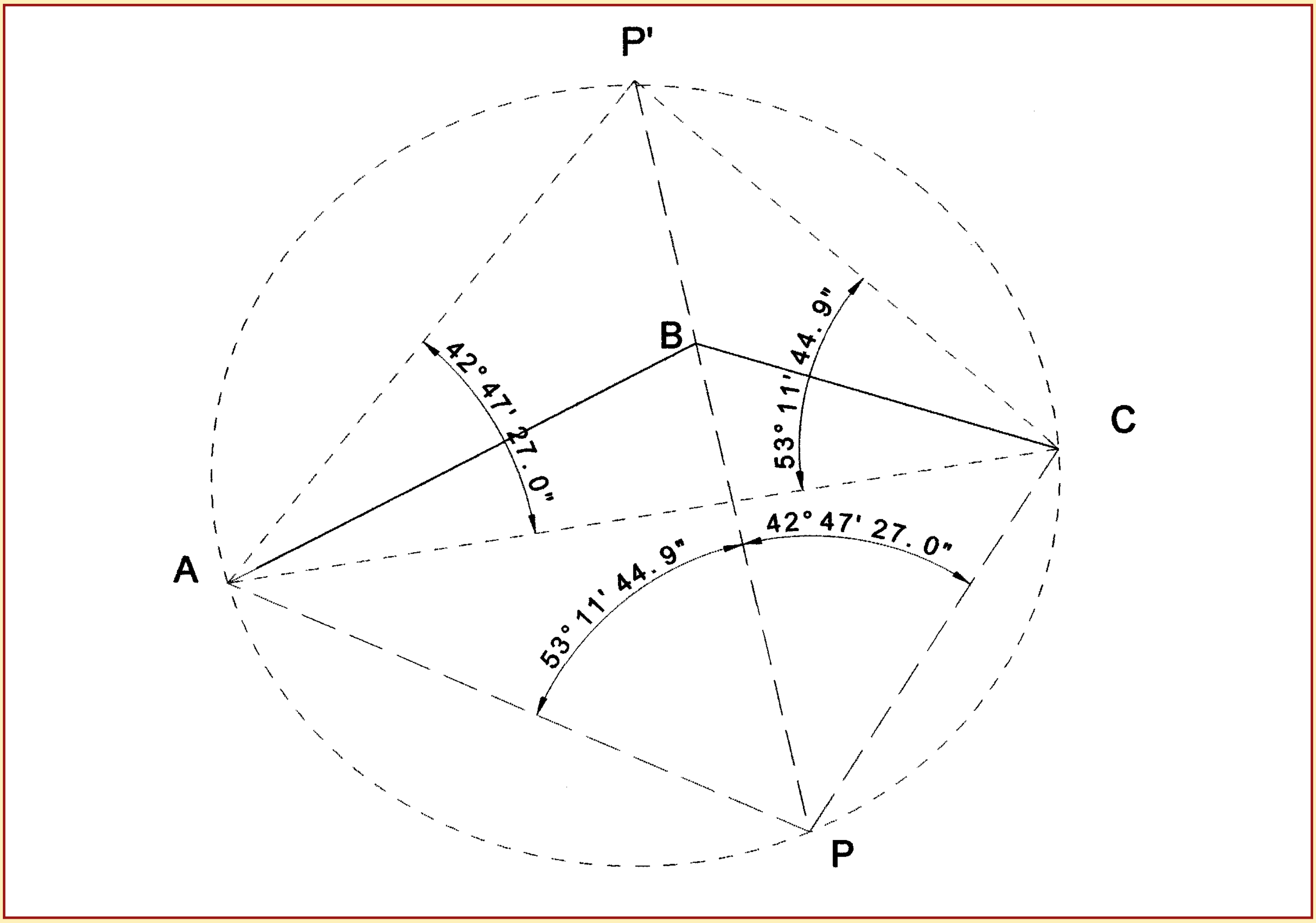
Solution # 1 to Problem 30



THIS SOLUTION IS KNOWN AS THE CASSINI AID:

CONSTRUCT A CIRCLE THROUGH A-B-P AND B-C-P. CONSTRUCT A-B' PERPENDICULAR TO A-B AND C-A' PERPENDICULAR TO B-C. DRAW B-B', B-A' AND B'-A'. POINT P LIES ON A LINE JOINING B' AND A' AND PB IS PERPENDICULAR TO A'-B'. ANGLE AB'B EQUALS ANGLE APB AND ANGLE CA'B EQUALS ANGLE CPB. TRIANGLES B-A-B' AND B-C-A' CAN EASILY BE SOLVED BY THE LAW OF SINES OR THE PYTHAGOREAN THEOREM, FROM WHENCE TRIANGLES B-B'-P AND B-A'-P CAN ALSO BE SOLVED. (SIDE B'-A' BY LAW OF COSINES, ANGLES B-B'-A' AND B-A'-B' BY LAW OF SINES)

Solution # 2 to Problem 30



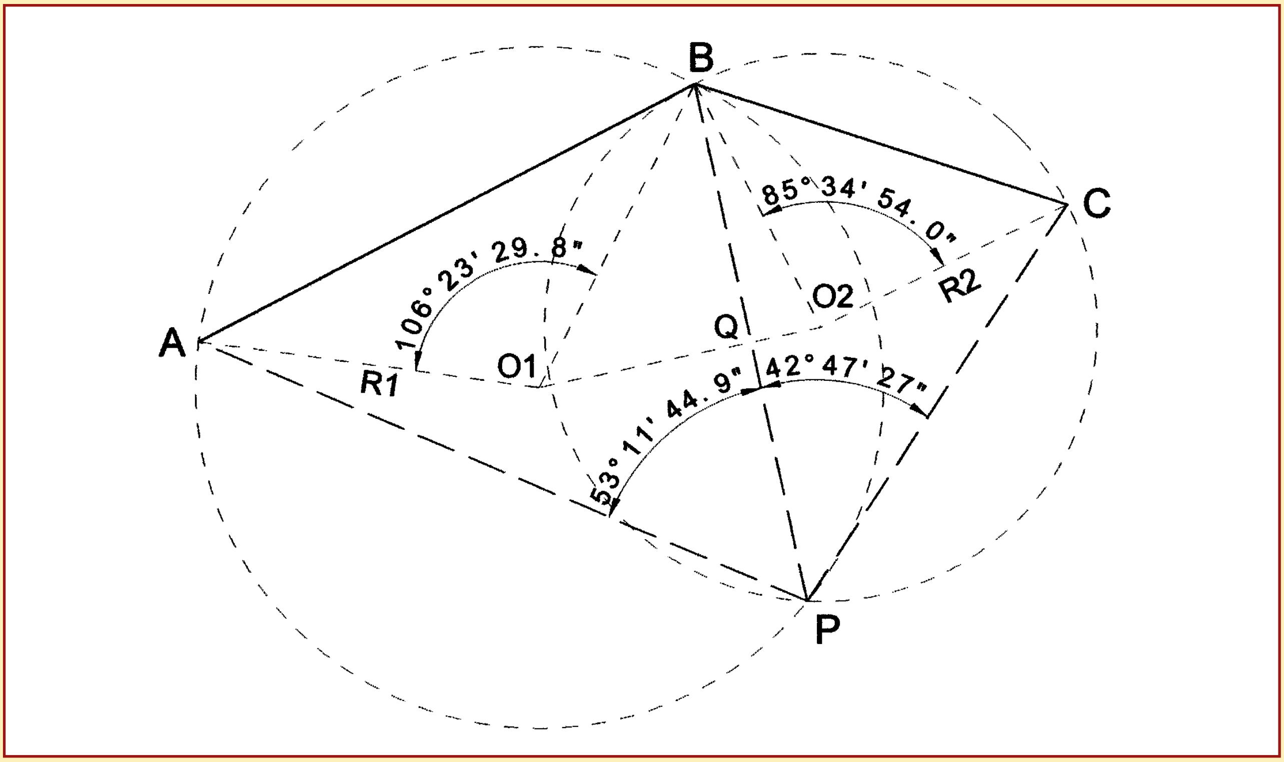
THIS SOLUTION IS KNOWN AS AN ITALIAN SECTION:

CONSTRUCT A CIRCLE THROUGH A-P-C. EXTEND P-B TO INTERSECT THE CIRCLE AT P'.

ANGLE APB IS EQUAL TO ANGLE ACP' AND ANGLE BPC IS EQUAL TO ANGLE CAP'.

AN INVERSE FROM A TO C YIELDS A BEARING AND DISTANCE FROM WHICH P' CAN BE SOLVED BY THE LAW OF SINES AND COORDINATED. AN INVERSE FROM B TO P' YIELDS THE BEARING OF LINE P-B, FROM WHICH BEARINGS OF A-P AND C-P CAN BE CALCULATED AND TRIANGLE A-C-P CAN BE SOLVED BY LAW OF SINES OR BY A BEARING-BEARING INTERSECTION TO GET COORDINATES OF POINT P.

Solution # 3 to Problem 30



THIS IS A STANDARD 3 POINT RESECTION PROBLEM:

CONSTRUCT A CIRCLE THROUGH A-B-P AND B-C-P. DRAW RADII O1-A, 01-B, 02-B, AND 02-C. DRAW 01-02, WHICH BISECTS P-B AT Q AND IS PERPENDICULAR TO P-B.

SINCE ANGLE A-O1-B IS TWICE ANGLE A-P-B AND ANGLE B-O2-C IS TWICE ANGLE B-P-C, ISOSCELES TRIANGLES A-01-B AND B-02-C CAN BE SOLVED FROM THE KNOWN (OR CALCULATED BY INVERSE) DISTANCES BETWEEN A-B AND B-C.

COORDINATES CAN BE COMPUTED FOR 01 AND 02 AND INVERSED FOR THE BEARING OF 01-02, OR TRIANGLE 01-B-02 CAN BE SOLVED AND BROKEN DOWN INTO TWO RIGHT TRIANGLES FOR THE BEARING OF B-Q, AND THEREFORE B-P, FROM WHICH BEARINGS A-P AND P-C CAN BE DERIVED.

SOLVE TRIANGLE A-B-P FOR DISTANCE A-P AND/OR TRIANGLE B-C-P FOR DISTANCE C-P TO COORDINATE POINT P.