To solve for the circle through vertex B, construct square ACed on side AC. Extend side DA to f and side eC to g so fg is parallel with AC and passes through B. Draw Bd and Be intersecting AC at h and i. Construct perpendiculars to AC at h and i, intersecting AB at j and BC at k. hijk forms a square and the circle through B passes through j and k also. (See Problem No. 114)

By the Law of Cosines calculate the angles of triangle ABC: angle A = 57°07’18”, angle B = 78°27’47” and angle C = 44°24’55”.

\[\begin{align*}
Af &= 500 \cdot \cos(90° - 57°07’18”) = 419.912, \\
fb &= 500 \cdot \sin(90° - 57°07’18”) = 271.428, \\
gc &= 600 \cdot \cos(90° - 44°24’55”) = 419.912 \quad \text{and} \\
bg &= 600 \cdot \sin(90° - 44°24’55”) = 428.572.
\end{align*}\]

Angle fDB = \(\arctan\frac{271.428}{119.912}\) = 13°37’26” and Ah = 700 \cdot \tan13°37’26” = 169.656

Angle geB = \(\arctan\frac{428.572}{119.912}\) = 20°56’28” and iC = 700 \cdot \tan20°56’28” = 267.878

(Ah and iC can be found by similar triangles too.)

Hi is therefore 700.000 - 169.656 - 267.878 = 262.466, which is equal to jk, a chord of the circle through point B. Angle jQk equals twice angle jBk = 156°55’34”. In triangle jQK, by the Law of Cosines, \(r^2 + r^2 - 2 \cdot r \cdot r \cdot \cos156°55’34” = 262.466^2\), and \(r = 133.939\)

(Alternatively, draw jm, a diameter of the circle. Angle jmk = angle jBk = 78°27’47” because they subtend the same chord. \(\frac{jk}{jm} = \sin\angle jmk = \frac{jk}{2r}\) and \(r = 133.939\).)

Likewise construct a square on side BC and use the same relative steps to find the radius of the circle through point A: 160.566

Finally, construct a square on side AB and use the same relative steps to find the radius of the circle through point C: 193.037