The values of C and D are readily calculated, first by finding the coordinate of the radius point, R, which is the midpoint of AB and the average of its coordinates: (−1.0, −1.5).

The radius is then $r = \sqrt{(1+1.5)^2 + 1.0^2} = \sqrt{7.75} = 2.69258$

(Alternatively it is $\frac{1}{2}\sqrt{(1+4)^2 + 2^2} = \sqrt{29/2} = 2.69258\)  

Construct a perpendicular from R to the x-axis at point M. RM=1.5.

$CM = MD = \sqrt{2.69258^2 - 1.5^2} = \sqrt{5} = 2.23607$

$C = -1.0 - \sqrt{5} = -3.23607, D = -1.0 + \sqrt{5} = 1.23607$

The general formula for a circle centered at (h, k) is $(x - h)^2 + (y - k)^2 = r^2$; for this circle it is $(x + 1)^2 + (y + 1.5)^2 = r^2 = 7.25$

When $y = 0$, $x^2 + 2x + 1 + 2.25 - 7.25 = 0$, or $x^2 + 2x - 4 = 0$

From the quadratic equation: $x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 4}}{2} = \frac{-2 \pm \sqrt{20}}{2} = -1 \pm \sqrt{5}$

The x values of C and D are therefore the roots of the quadratic equation $x^2 + 2x - 4 = 0$, or $x^2 - (-2)x + (-4) = 0$. Note the values (-2, -4) of point B.

In any quadratic equation of the form $x^2 - sx + p = 0$, a circle diameter through (0, 1) and (s, p) is called the Carlyle Circle of the equation. Where the circle crosses the x-axis is/are the root(s) of the equation. It is the CONSTRUCTION of the roots.

The center of the circle is also the midpoint of the line joining (s, 0) and (0, p+1), or \[ \left( \frac{s + p + 1}{2} \right) \]

If m and n are the roots of the equation, it may be written as $x^2 - sx + p = (x - m)(x - n) = x^2 - (m + n)x + mn$

so that $s = (m + n)$ and $p = mn$. (These are known as Vieta’s equations.)

In this case, $s = (-1 + \sqrt{5}) + (-1 - \sqrt{5}) = -2$ and $p = (-1 + \sqrt{5})(-1 - \sqrt{5}) = -4$