PROBLEM NO. 192 SOLUTION

By the Law of Cosines:

Angle A = 57°07'18", angle B = 78°27'47", and angle C = 44°24'55"

Angle C-A-A' = angle A'-A-B = 28°33'39"

Angle A-B-B' = angle B'-B-C = 39°13'53.5"

Angle B-C-C' = angle C'-C-A = 22°12'27.5"

Bisectors divide the intersected side into parts proportional to the other sides:

\[ AC' = \frac{700}{500}, \quad AC'' = \frac{600}{500}, \quad C'B = 230.7692, \]

\[ BA' = \frac{500}{600}, \quad BA'' = \frac{250.000}{600}, \quad A'C = \frac{700}{500}, \quad A''C = 350.000, \]

\[ AB' = \frac{500}{700}, \quad AB'' = \frac{381.8182}{700}, \quad B'C = \frac{500}{700}, \quad B''C = 381.8182 \]

Letting point A be North 0, East 0, and line AC = East, point C is North 0, East 700 and point B, by bearing-bearing, azimuth-azimuth or distance-distance intersection is North 419.9125, East 271.4286.

By traversing or proportional parts, C’ is North 226.1067, East 146.1537, A’ is North 244.9488, East 450.0000, and B’ is North 0, East 318.1818.

The radius point of the circle is NOT the intersection of the triangle bisectors. C'A', A'B' and B'C' are chords of the circle. Their perpendicular bisectors intersect at the center of the circle.

The coordinate of the midpoint of any line is the average of the end coordinates, so that "a" is North 113.0534, East 232.1678, point "b" is North 235.5278, East 298.0768, and point "c" is North 122.4744, East 384.0909. The bearing of B'C' is N 37°15'53.5" W, B'A' is N 28°17'12.2" E, and C'A' is N 86°27'05.5" E. Using points "a" and "c", and calling the radius point Q, the bearing of "ac" is N 86°27'05.5" E (same as C'A') and the perpendicular bisectors at "a" and "c" are N 12°52'06.5" E and N 61°42'47.8 W, respectively. A sketch will show:

\[ \frac{aQ}{\sin 31°50'06.7"} = \frac{cQ}{\sin 33°42'59.0"} = \frac{152.2149}{\sin 112°42'54.9"} \]

from which aQ = 88.1983 and cQ = 92.8140.

so that point Q is North 166.4575, East 302.3601.

QA' = N 62°00'11" E 167.2077, QB' = S 5°25'47" E 167.2077 and N 69°06'00" w QC' = 167.2078 = radius of circle.

In triangle QC'd, angle dC'Q = 78°01'18", angle C'Qd = 23°57'24" and chord C'd = 69.4056.

In triangle QA'e, angle QA'e = 72°24'44 angle A'Qe = 35°10'32" and chord A'e = 101.0494.

In triangle B'Qf, angle fB'Q = 84°34'13", angle B'Qf = 105°51'34" and chord B'f = 31.6434.

Except for round-off errors, chord A'e = chord B'f + chord C'd.