Solution to Problem 95

There is no direct solution to this problem, but it is easily solved with a few trials. Let the instrument set-up point have northing and easting of zero. Let the line to the E.C. be "North". Calculate the coordinate of the B.C., as shown. The bearing to the radius point will be N 71°27' E. The solution is found when a traverse from the B.C. through the radius point to the E.C. yields a zero value for the easting of the E.C. The radius is dependent on φ, the central angle (87.24' = R.φ, with φ in radians). Since no value for φ is easily ascertained a table is constructed of possible values:

<table>
<thead>
<tr>
<th>φ</th>
<th>R</th>
<th>East'g R.P.</th>
<th>East'g E.C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>40°</td>
<td>124.96'</td>
<td>90.85</td>
<td>-25.45</td>
</tr>
<tr>
<td>50°</td>
<td>99.97'</td>
<td>67.16</td>
<td>-18.12</td>
</tr>
<tr>
<td>60°</td>
<td>83.31'</td>
<td>51.37</td>
<td>-11.08</td>
</tr>
<tr>
<td>70°</td>
<td>71.41'</td>
<td>40.09</td>
<td>-4.42</td>
</tr>
<tr>
<td>80°</td>
<td>62.48'</td>
<td>31.62</td>
<td>1.76</td>
</tr>
</tbody>
</table>

The answer appears between 70° and 80°. Straight interpolation gives 77° 09' 08" as an answer:

77°09'08"  64.787'  33.807  0.056

This is close, but not quite correct. Using 77°:

77°  64.915'  33.929  -0.038

Now using 77° and 77° 09' 08" for interpolation yields 77°03'43"

77° 03'43"  64.863'  33.880  0.00003, close enough!

The central angle is 77° 03' 43" and the radius is 64.86'

(This problem is a variation of one presented by Ty Clinghouse of Oklahoma and the solution is an easier one than mine provided by John Nolton of Arizona.)
Solution to Problem 96

Let $N_1$ and $E_1$ be the coordinates of point 1, $N_2$ and $E_2$ the coordinates of point 2 and $N_0$ and $E_0$ the coordinates of the intersection point. The azimuth from point 1 is $A_1$ and the azimuth from point 2 is $A_2$.

$$E_0 = E_1 + (N_0 - N_1) \tan A_1 = E_2 + (N_0 - N_2) \tan A_2$$

$$E_1 + N_0 \tan A_1 - N_1 \tan A_1 = E_2 + N_0 \tan A_2 - N_2 \tan A_2$$

$$E_1 - E_2 - N_1 \tan A_1 + N_2 \tan A_2 = N_0 \tan A_2 - N_0 \tan A_1$$

$$N_0 = \frac{E_1 - E_2 - N_1 \tan A_1 + N_2 \tan A_2}{\tan A_2 - \tan A_1}$$

(All of the quantities on the right side of the equation are known)

$$E_0 = E_1 + (N_0 - N_1) \tan A_1$$

Note that 90° and 270° azimuths cannot be used! (Use 89°59′59″ or 90°00′01″ or 269°59′59″ or 270°00′01″ which will work for most surveying applications)

It is highly unlikely the denominator for $N_0$ will ever be zero: the azimuths would have to be 180° apart, not impossible but not very likely.

Of the ten possible combinations of azimuths (both <90°; 90°>A_1 or A_2 and 90° < A_2 or A_1 < 180°; A_1 or A_2 < 90° and 180° < A_2 or A_1 < 270°; A_1 or A_2 < 90° and 270° > A_2 or A_1 > 180°; 90° < both <180°; 180° < A_1 or A_2 < 270° and 90° < A_2 or A_1 < 180°; 90° < A_1 or A_2 < 180° and 270° < A_2 or A_1 < 360°; 180° < both <270°; 180° < A_1 or A_2 < 270° and 270° < A_2 or A_1 < 360°; 270° < both <360°), the position of the intersection point is determined by the algebraic signs of the tangents of the azimuths. If you are doing this by hand be careful of the algebraic sign.