Solution
168
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By difference of bearings, angle BAD = 45°22'15".

Using the Law of Cosines, BD² = AB² + AD² - 2 AB AD cos 45°22'15", so BD = 83.1219

In triangle BQD, BQ = QD = radius. Angle BQD = 90°44'30" (twice angle BAD).

Chord BD = 2 R sin 45°22'15", so that R = 58.3993'

In triangle AQB by the Law of Cosines, 92.174² = R² + R² - 2 R² cos angle AQB, so that angle AQB = 104°12'58".

Angle AQB + angle BQC = 104°12'58" + 45°22'15" = 149°35'13"

Chord AC = 2(58.3993) sin[½(149°35'13")] = 112.709'

(Note: This is a weak solution, using cosines of angles between 120° & 180°.)

Alternatively,

Extend line AD to intersect CA', where CA' = AC. Construct AF = AB.

AA' = 2 AC cosφ = AD + DA', so that 2AC cosφ - AD = DA'

AE - ½ FD = AF, and FD = AD - AF

2 AE - FD = 2 AF, and 2AE - (AD - AF) = 2 AF, and 2 AE - AD = AF

but AE = 2 AC cosφ, so DA' = AF

Therefore, 2 AC cosφ = AD + AF, but AF = AB

so, 2 AC cosφ = AD + AB, or AC = (AD + AB)/ 2 cos φ

(This is known as the Three Chord Lemma)
Solution to Problem 169

a) Here x = 2, and n = 15. (15-2)/6 = 2 \ R \ 1, so that SDQ(2) = 2, and R = 1. From the table the answer is 8. Thus, SDQ(2^{15}) = 8, and since SDQ(32,868) = 27 \Rightarrow 9 this answer must be incorrect. 
Correct answer: (32,768) \Rightarrow 26 \Rightarrow 8.

b) \ 16^7 =? \ 268,335,456.
SDQ(16) = 7. For n = 7 we need not use the remainder calculation. From the table the answer is 7.
SDQ(268,335,456) = 42 \Rightarrow 8. So this answer is incorrect.
Correct answer: (268,435,456) \Rightarrow 43 \Rightarrow 7.

c) \ 823 =? \ 590,295,810,358,305,651,712 \Rightarrow 85 \Rightarrow 13 \Rightarrow 4.
\frac{(23 - 2)}{6} = 3 \ R = 3
From the table we get 8. So this answer is incorrect.
Correct answer: (590,295,810,358,705,651,712) \Rightarrow 89 \Rightarrow 17 \Rightarrow 8.

d) \ 147^{28} =? \ 4,840,445,926,998,527,143,180,132,566,802,461,408,607,116,960,093,883,732,904,561\Rightarrow 263 \Rightarrow 11
\Rightarrow 2
SDQ(147) = 12 \Rightarrow 3
\frac{(28-2)}{6} = 4 \ R = 2
From the table the answer is 9. Therefore the answer given is incorrect.
\Rightarrow 9.