Solution to Problem 119

Let quadrilateral A-B-C-D be a cyclic quadrilateral (inscribed in a circle).

Construct angle A-B-E = angle D-B-C.

Angle C-A-B = angle C-D-B because they subtend a common chord, BC.

Triangle ABE is therefore similar to triangle DBC, with all angles equal,

so, \( \frac{AE}{AB} = \frac{DC}{DB} \) and \( AE \times DB = AB \times DC \) ..............................................................(1)

Angle A-B-E = angle D-B-C (by construction).

Angle A-B-E + angle E-B-D = angle D-B-C + angle E-B-D,

But angle A-D-B = angle B-C-E because they subtend the common chord AB.

Triangle ABD is therefore similar to triangle EBC,

so \( \frac{AD}{DB} = \frac{EC}{CB} \) and \( EC \times DB = AD \times CB \) ..............................................................(2)

Adding (1) and (2), \( AE \times DB + EC \times DB = AB \times DC + AD \times CB \).

\( (AE + EC) \times DB = AB \times DC + AD \times CB \)

but \( AE + EC = AC \)

so, for a quadrilateral to be inscribed in a circle, \( AC \times DB = AB \times DC + AD \times CB \).

For this problem, AC=723.3527, DB=755.3010, AB=337.8837, DC=587.5896, AD=639.0341 and CB=544.2778

Does 723.3527 \times 755.3010 = 337.8837 \times 587.5896 + 639.0341 \times 544.2778 ?

546,349.0177 = 198,536.9481 + 347,812.0741

546,349.0177 = 546,349.0222

Close enough!
Solution to Problem 120

\[ X^2 = 2A \]
\[ Y^2 = A \]
\[ X^2 = 2Y^2 \]
\[ X = \sqrt{2}Y \]

3 \( X + X - Y + 3Y = 5280' \)
4 \( X + 2Y = 5280' \)
2 \( X + Y = 2640' \)

2 \( \sqrt{2}Y + Y = 2640' \)
\( (1 + 2\sqrt{2})Y = 2640' \)
\( Y = 2640'/3.828427125 = 689.578' \)

\( X = (\sqrt{2})(689.578) = 975.211' \)

NOTE THAT \( X^2 = 951,036.27 \text{ ft}^2 \) AND \( Y^2 = 475,518.135 \text{ ft}^2 \)