Solution to Problem 113

Let square PQRS be 1 mile on a side.

1. Area $A + 4 \times \text{area } B + 4 \times \text{area } C = 1$ square mile.
2. Area $A + 3 \times \text{area } B + 2 \times \text{area } C = \frac{\pi}{4}$ of a circle = $\pi/4$
3. Area $A + 2 \times \text{area } B + \text{area } C = 2 \times \text{area } E + \text{area } D$

Triangle QTR is equilateral and has an area of $\sqrt{3}/4$

$2 \text{ area } E + \text{area } D = \text{Sector } TQR + \text{sector } QRT - \triangle QTR$

Sector TQR = Sector QRT = $\pi/6$

1. $A + 4B + 4C = 1$
2. $A + 3B + 2C = \pi/4$
3. $A + 2b + C = \pi/3 - \sqrt{3}/4$

Combining (1) and (2), $B + 2C = 1 - \pi/4$ (4)

Combining (2) and (3), $B + C = \pi/4 - \pi/3 + \sqrt{3}/4$ (5)

Combining (4) and (5), $C = 1 - \sqrt{3}/4 - \pi/6 = 0.043388523$ sq. mi.

By substitution of $C$ into (5), $B + 1 - \sqrt{3}/4 - \pi/6 = \pi/4 - \pi/3 + \sqrt{3}/4$

and $B = \pi/12 + \sqrt{3}/2 - 1 = 0.127824792$ sq. mi.

By substitution of $B$ and $C$ into (2),

$A + 3(\pi/12 + \sqrt{3}/2 - 1) + 2(1 - \sqrt{3}/4 - \pi/6)$

And $A = 1 + \pi/3 - \sqrt{3} = 0.315146744$ sq. mi.
Solution to Problem 114

First, solve for the angles in the triangle by the Law of Cosines:

\[ A = 70°29'02.4'' \], \[ B = 71°44'11.5'' \], \[ C = 37°46'46.1'' \]

Construct square A-C-R-S on side AC. Construct lines BS and BR, intersecting AC at P and Q. Construct perpendiculars to AC at P and Q. The intersections with sides AB and BC are the corners of the square as are points P and Q.

Let AC bear East. Traverse from A to B (or C to B, or both for a check) using the complements of the angles calculated for the triangle as bearings. Traverse to S and R, or just assign coordinates to them. Inverse from S to B and from R to B.

SB = N 7°38'00'' E, RB = N 26°00'19'' W (No distances are needed)

\[ AP = 5764.40 \times \tan 7°38'00'' = 772.56' \]

\[ CQ = 5764.40 \times \tan 26°00'19'' = 2812.14' \]

\[ PQ = 5764.40 - 772.56 - 2812.14 = 2179.70' \]

Now, do the same for the other two sides: (Just substitute A' or A'' for A, etc.)

The bearing for S'B' is N 26°12'55'' E and for R'B' is N 7°10'43'' W
\[ A'P' = 2817.195', C'Q' = 720.62', \text{ making } P'Q' = 2183.625' \]

The bearing for S"B" is N 11°05'05'' E and for R"B" is N 11°52'57'' W
\[ A"P" = 728.565', C"Q" = 782.48', \text{ making } P"Q" = 2207.695' \]

The square constructed on the shortest side is the largest.