Solution to Problem 75

You can find the lengths of the sides from Problem No. 54, or calculate them:
The area of the triangle = 1 acre = 43,560 sq. ft. = \( \frac{1}{2} \) ABxAC, but
AB = 0.75 AC, so \( 2 \times (43,560) = (0.75 AC)(AC) \), or 87120 = 0.75AC^2
from which AC = 340.822’ and AB = 255.617’ and BC = 426.028’.

The length of the rope must be 213.014’ + 340.822” - x (going counter-
clockwise) and 213.014’ + 255.617’ + x (going clockwise), from which x =
42.602’ and the length is 511.234’.

Alternatively, twice the length must be 340.822’+426.028’+255.617’ (the
perimeter of the triangle) and the length is 511.234’.

The central angle of Area I is 180° – arctan 0.75 = 180° – 36°52’12” or 143°07’48”

The central angle of Area III is 180° – arctan 1.33333... = 180° – 53°07’48” or
126°52’12”

The grazing area consists of four parts;

Area I = \( \frac{143°07’48”}{360°} \times \pi \times 298.22^2 = 111,084.02 \text{ sq. ft.} \)

Area II = \( \frac{1}{2} \times \pi \times 511.234^2 = 410,543.65 \text{ sq. ft.} \)

Area III = \( \frac{126°52’12”}{360°} \times \pi \times 298.22^2 = 98,464.54 \text{ sq. ft.} \)

Area IV = \( \frac{1}{4} \times \pi \times 42.602^2 = 1,425.44 \text{ sq. ft.} \)

for a total of 621,517.64 sq. ft. or 14.268 Acres.
Solution to Problem 76
Case 1

(CASE 1) WHERE THE LATITUDE IS GREATER THAN THE DECLINATION AND THE SHADOWS ARE NORTH OF THE FLAGPOLE:

Let $\phi =$ latitude and $\delta =$ declination and $H =$ height of flagpole.

From the drawing, $H \tan (\phi + \delta) = 8 H (\phi - \delta)$ or

$$\tan (\phi + 23^\circ 28') = 8 \tan (\phi - 23^\circ 28').$$

Equating by tangent of sums,

$$\frac{\tan \phi + \tan 23^\circ 28'}{1 - \tan \phi \tan 23^\circ 28'} = 8 \left( \tan \phi - \tan 23^\circ 28' \right)$$

Expanding and rearranging yields $9K \tan^2 \phi - 7(1 + K^2) \tan \phi + 9K = 0$,

where $K = \tan 23^\circ 28' = 0.434120782$, from which $\phi = 55^\circ 02'01''$ and $34^\circ 57'59''$
Solution to Problem 76
Case 2

(CASE II) WHERE THE LATITUDE IS LESS THAN THE DECLINATION AND ONE
SHADOW IS NORTH OF THE FLAGPOLE AND THE OTHER IS SOUTH:

\[ 8 \cdot H \cdot \tan (\delta - \phi) = H \cdot \tan (\phi + \delta) \]

\[ 8 \cdot \tan (23^\circ 28' - \phi) = \tan (\phi + 23^\circ 28') \]  Equating to tangents of sums,

\[ \frac{8 \cdot (\tan 23^\circ 28' - \tan \phi)}{1 + \tan 23^\circ 28' \cdot \tan \phi} = \frac{\tan \phi + \tan 23^\circ 28'}{1 - \tan \phi \cdot \tan 23^\circ 28'} \]

Expanding and rearranging yields

\[ 7K \cdot \tan^2 \phi - 9(K^2 + 1) \cdot \tan \phi + 7K = 0, \]  where \( K = \tan 23^\circ 28' = 0.434120782, \)

from which \( \phi = 17^\circ 18'46'' \) (The other solution, \( 72^\circ 41'13'' \), gives a physically
impossible answer as the altitude of the sun would be \(-6^\circ 09''\))